

库仑定律 $F_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \vec{r}_{12}$ $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2$

电场 $E = \frac{F}{q_0}$ (定义式) 点电荷 $E = \frac{q}{4\pi\epsilon_0 r^2} \vec{r}$

场强叠加原理. 矢量和.

电通量 $\Phi_e = \int_S \vec{E} \cdot d\vec{S}$ eg 均匀电场与平面法线夹角 θ , $\Phi = ES \cos \theta$.

高斯定理: 闭合曲面电通量等于其包含的电荷量代数和除以 ϵ_0 .

$$\Phi_e = \oint_S \vec{E} \cdot d\vec{S} = \frac{1}{\epsilon_0} \sum q_i.$$

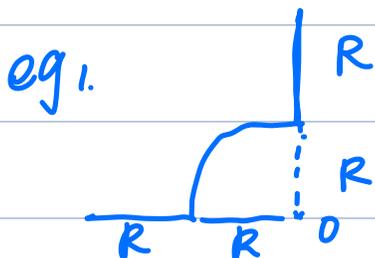
一些微元: 直线 dl , 圆弧 $R d\theta$, 圆环 $2\pi r dr$, 球壳 $4\pi r^2 dr$.

电荷密度 线 λ , 面 σ , 体 ρ .

电荷微元 dq . $dq_1 = \lambda dl$, $dq_2 = \sigma dS$, $dq_3 = \rho dV$.

$$E = \frac{q}{4\pi\epsilon_0 r^2} \vec{r} \quad dE = \frac{dq}{4\pi\epsilon_0 r^2} \vec{r}$$

$$\vec{E} = \int dE = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2} \vec{r}. \quad \text{注意不同方向分量.}$$



线密度 λ 求 O 处场强.

$$E_1 = \frac{\lambda}{4\pi\epsilon_0} \int_R^{2R} \frac{dl}{l^2} = \frac{\lambda}{8\pi\epsilon_0 R} \quad (\text{向下}).$$

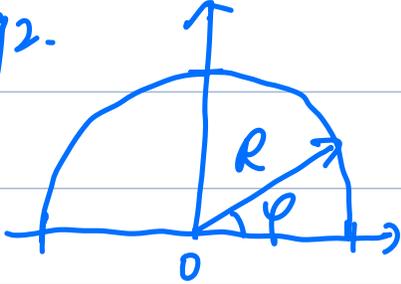
$$E_2 = \frac{\lambda}{8\pi\epsilon_0 R} \quad (\text{向右}).$$

$$d\vec{E}_2 = \frac{\cos\theta dq}{4\pi\epsilon_0 R^2} = \frac{\cos\theta \lambda R d\theta}{4\pi\epsilon_0 R^2} \quad E_2 = \frac{\lambda}{4\pi\epsilon_0 R} \int_{-\pi/4}^{\pi/4} \cos\theta d\theta.$$

$$E_3 = \frac{N\lambda}{4\pi\epsilon_0 R} \quad (\text{向右下 } 45^\circ).$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \frac{3N\lambda}{8\pi\epsilon_0 R} \quad (\text{向右下 } 45^\circ).$$

eg2.



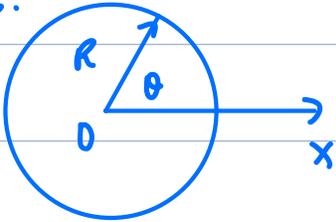
线密度 $\lambda = \lambda_0 \sin\varphi$. 求 O 处场强.

关于 y 轴对称.

$$dE = \frac{\sin\varphi dq}{4\pi\epsilon_0 R^2} = \frac{\lambda_0 \sin^2\varphi d\varphi}{4\pi\epsilon_0 R}.$$

$$E = \frac{\lambda_0}{4\pi\epsilon_0 R} \int_0^\pi \frac{1 - \cos 2\varphi}{2} d\varphi = \frac{\lambda_0}{8\epsilon_0 R} \quad (\text{竖直向下}).$$

eg3.



球面带电电荷面密度 $\sigma = \sigma_0 \cos\theta$.

求球心处电场强度 E .

关于 x 轴对称.

$$dE = \frac{\cos\theta dq}{4\pi\epsilon_0 R^2}, \quad dq = \sigma_0 \cos\theta dS = \sigma_0 \cos\theta \cdot R \sin\theta \cdot 2\pi \cdot R d\theta.$$

$$\therefore dE = \frac{\sigma_0 \cos^2\theta \sin\theta}{2\epsilon_0} d\theta \quad E = \frac{\sigma_0}{2\epsilon_0} \int_0^\pi \cos^2\theta \sin\theta d\theta.$$

$$\int_0^\pi (1 - \sin^2\theta) \sin\theta d\theta = \int_0^\pi \sin\theta d\theta - \int_0^\pi \sin^3\theta d\theta.$$

$$= \int_0^\pi \sin\theta d\theta + \int_0^\pi (1 - \cos^2\theta) d(\cos\theta) = 2 - \int_{-1}^1 (1 - t^2) dt = \frac{2}{3}.$$

$$\therefore E = \frac{\sigma_0}{2\epsilon_0} \cdot \frac{2}{3} = \frac{\sigma_0}{3\epsilon_0} \quad \text{向 } x \text{ 轴负方向}.$$

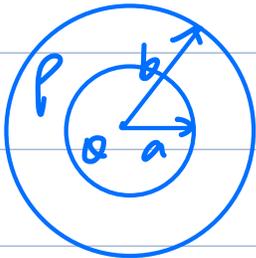
高斯定理使用 ① 找特定闭合曲面 (高斯面).

面上各点场强大小相同. 面与场垂直.

① 设场强 E , 用定义求出电通量 Φ .

② 用高斯定理求 Φ . 解 E .

eg4



带电球壳内外半径为 a, b .

电荷体密度 $\rho = A/r$. r 为离球心距离

球心处有一点电荷 Q . 求 E 的分布.

$$4\pi r^2 E = \frac{1}{\epsilon_0} \Sigma q \quad E = \frac{1}{4\pi\epsilon_0 r^2} \Sigma q.$$

$$r < a \quad \Sigma q = Q \quad E = \frac{Q}{4\pi\epsilon_0 r^2}.$$

$$a < r < b \quad \Sigma q = Q + \int \rho dV = Q + A \int_a^r \frac{4\pi r^2}{r} dr$$

$$= Q + 2\pi A (r^2 - a^2)$$

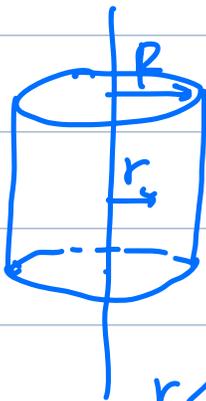
$$E = \frac{Q + 2\pi A (r^2 - a^2)}{4\pi\epsilon_0 r^2}.$$

$$r > b \quad \Sigma q = Q + \int \rho dV = Q + A \int_a^b \frac{4\pi r^2}{r} dr.$$

$$= Q + 2\pi A (b^2 - a^2).$$

$$E = \frac{Q + 2\pi A (b^2 - a^2)}{4\pi\epsilon_0 r^2}.$$

eg5.



半径 R 的无限长圆柱 $\rho = ar - br^2$.

求其内外电场分布.

$$2\pi r \cdot h \cdot E = \frac{1}{\epsilon_0} \Sigma q.$$

$$r < R \quad \Sigma q = \int_0^r (ar - br^2) \cdot 2\pi r \cdot h \cdot dr.$$

$$= 2\pi h \cdot \left(\frac{1}{2} ar^3 - \frac{1}{4} br^4 \right)$$

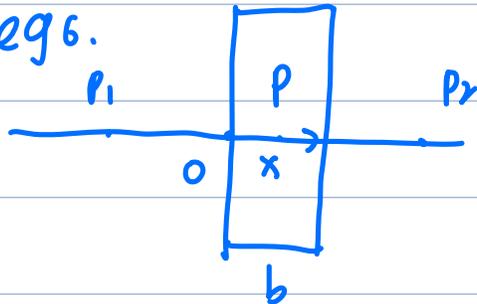
$$\therefore E = \frac{1}{\epsilon_0} \left(\frac{1}{2} ar^2 - \frac{1}{4} br^3 \right)$$

$$r > R \quad \Sigma q = \int_0^R (ar - br^2) \cdot 2\pi r \cdot h \cdot dr.$$

$$= 2\pi h \left(\frac{1}{3} aR^3 - \frac{1}{4} bR^4 \right),$$

$$\therefore E = \frac{1}{\epsilon_0 r} \left(\frac{1}{3} aR^3 - \frac{1}{4} bR^4 \right).$$

eg 6.

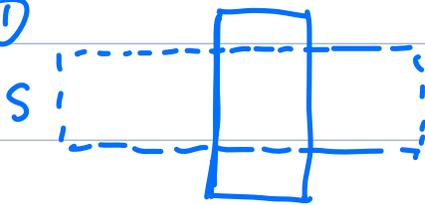


厚为 b 的无限大带电平板.

$$\rho = kx^2 \quad (0 \leq x \leq b)$$

求平板两侧及内部电场大小.

①

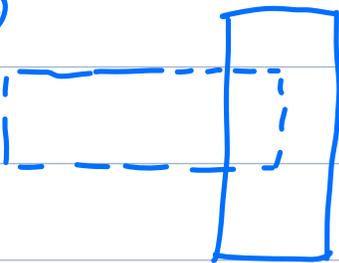


$$2SE = \frac{1}{\epsilon_0} \Sigma q = \frac{1}{\epsilon_0} \int_0^b kx^2 \cdot S dx$$

$$\therefore 2SE = \frac{Sk}{\epsilon_0} \frac{1}{3} b^3.$$

$$E = \frac{kb^3}{6\epsilon_0}.$$

②



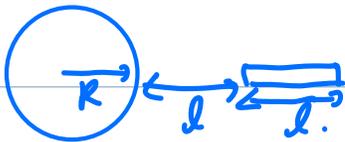
$$S(E + E') = \frac{1}{\epsilon_0} \Sigma q = \frac{S}{\epsilon_0} \int_0^{x_0} kx^2 dx.$$

$$\therefore S(E + E') = \frac{Sk}{3\epsilon_0} x_0^3.$$

$$\therefore E' = \frac{kx_0^3}{3\epsilon_0} - E = \frac{k(2x_0^3 - b^3)}{6\epsilon_0}.$$

eg 7.

金属球半径为 R . $\rho = kr^2$



长 l 电荷线密度 λ 的细棒如图摆放.

求细棒受的静电力.

$$E \cdot 4\pi r^2 = \frac{1}{\epsilon_0} \int_0^R kr^2 \cdot 4\pi r^2 dr = \frac{1}{\epsilon_0} \frac{4}{5} \pi kR^5$$

$$\therefore E = \frac{kR^5}{5\epsilon_0 r^2} \quad (\text{此处 } r > R).$$

$$dF = E dq = \frac{kR^5}{5\epsilon_0 r^2} \lambda dr \quad (\text{水平向右}).$$

$$\therefore F = \frac{kR^5 \lambda}{5\epsilon_0} \int_{R+l}^{R+l+l} \frac{1}{r^2} dr = \frac{kR^5 \lambda}{5\epsilon_0} \left(\frac{1}{R+l} - \frac{1}{R+l+l} \right)$$

高斯定理亦可用于反推电荷量.

eg 8. 一立方体高斯面 $E_x = bx$ $E_y = 0$ $E_z = 0$.

边长 $a = 0.1 \text{ m}$ $b = 1000 \text{ N}/(\text{C}\cdot\text{m})$.

求其包含的电荷量.

$$\Phi = b(2a-a) \cdot a^2 = a^3 b = \sum \Sigma q_i.$$

$$\therefore \Sigma q_i = \epsilon_0 \cdot a^3 b = 8.85 \times 10^{-12} \text{ C}.$$

均匀带电球面 (电量 q_0) $E = \frac{q_0}{4\pi\epsilon_0 r^2}$

无限大平板 (面密度 σ) $E = \frac{\sigma}{2\epsilon_0}$.

均匀带电圆柱面 (线密度 λ) $E = \frac{\lambda}{2\pi\epsilon_0 r}$.

电势. 静电场的环路定理.

① 电场力做功路径无关. 始末位置相关.

② 电场强度沿任意闭合回路线积分恒为 0 $\oint E dl = 0$.

* 说明静电场是保守场. 可引入势能概念.

电荷移动过程中电场力做的功等于静电势能增量的负值.

$$A_{ab} = \int_a^b q_0 \cdot E \cdot dl = -(W_b - W_a) = -\Delta W.$$

电势能是相对的, 通常规定无穷远处为 0 势能.

$$W_p = \int_p^\infty q_0 \cdot E \cdot dl.$$

电势是某点的电势能与电荷量的比值。

$$U_p = \frac{W_p}{q_0} = \int_p^{\infty} \vec{E} \cdot d\vec{l}$$

$$A_{ab} = q_0 \int_a^b \vec{E} \cdot d\vec{l} = q_0 (U_a - U_b)$$

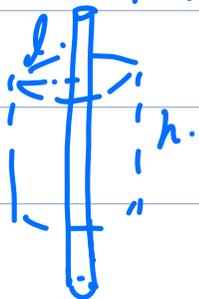
eg 10. 求半径 R 、总电量 Q 的均匀带电球面的电场中的电势分布。
(规定无穷远处电势为 0)。

$$\vec{E} = \begin{cases} \frac{Q}{4\pi\epsilon_0 r^2} \vec{r} & r > R \\ 0 & r < R \end{cases}$$

$$U_p = \int_p^{\infty} \vec{E} \cdot d\vec{l} = \int_r^{\infty} \frac{Q}{4\pi\epsilon_0 r^2} dr = \frac{Q}{4\pi\epsilon_0 r} \quad (r > R)$$

$$U_p = \int_p^{\infty} \vec{E} \cdot d\vec{l} = \int_r^R 0 \cdot dr + \int_R^{\infty} \frac{Q}{4\pi\epsilon_0 r^2} dr = \frac{Q}{4\pi\epsilon_0 R} \quad (r < R)$$

无限大均匀带电直线，电荷线密度 λ ，求垂直距离 r 处电势



高斯定理. $2\pi r \cdot h \cdot E = \frac{1}{\epsilon_0} (\lambda h) \quad \therefore E = \frac{\lambda}{2\pi r \epsilon_0}$

$$U = \int_r^{r_0} \vec{E} \cdot d\vec{l} = \frac{\lambda}{2\pi\epsilon_0} \int_r^{r_0} \frac{1}{l} dl = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{r_0}{r}$$

$\therefore r_0$ 为电势零点。

单个点电荷 q_i 的电势分布 $U = \frac{q_i}{4\pi\epsilon_0 r}$

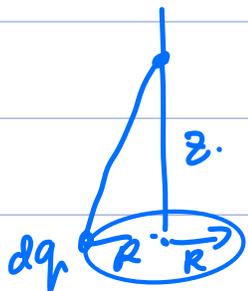
电势用电场的叠加来求解。

$$\begin{aligned} U_p &= \int_r^{+\infty} \vec{E} \cdot d\vec{l} = \int_r^{+\infty} (\vec{E}_1 + \vec{E}_2 + \dots + \vec{E}_n) \cdot d\vec{l} \\ &= \int_r^{+\infty} \vec{E}_1 \cdot d\vec{l} + \int_r^{+\infty} \vec{E}_2 \cdot d\vec{l} + \dots + \int_r^{+\infty} \vec{E}_n \cdot d\vec{l} \\ &= U_{p1} + U_{p2} + \dots + U_{pn} = \sum_{i=1}^n U_{pi} = \sum_{i=1}^n \frac{q_i}{4\pi\epsilon_0 r_i} \end{aligned}$$

可以分解成 dq , 计算出 dU . 再求解 U .

eg 11.

均匀带电圆环半径为 R . 带电量 q . 求轴线上一点的电势 (规定无穷远处电势为 0)



$$\lambda = \frac{q}{2\pi R}$$

$$dq = \lambda R d\theta = \frac{q}{2\pi} d\theta$$

$$dU = \frac{q}{8\pi^2 \epsilon_0 \sqrt{R^2 + z^2}} d\theta \quad U = \int_0^{2\pi} dU = \frac{q}{4\pi \epsilon_0 \sqrt{R^2 + z^2}}$$

球面 (即使不均匀) 带电 Q . 在球心处产生的电势

$$U = \int_S \frac{dq}{4\pi \epsilon_0 R} = \frac{1}{4\pi \epsilon_0 R} \int_S dq = \frac{Q}{4\pi \epsilon_0 R}$$

电场与电势梯度的关系. 场强矢量等于负的电势梯度矢量.

$$\vec{E} = -\text{grad } U = -\text{grad} \left(\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z} \right)$$

eg 12. 某区域内电势分布函数 $U = ax^2 + bxy - cz^2$.

求区域中任一点场强.

$$\vec{E} = -\text{grad } U. \quad \frac{\partial U}{\partial x} = 2ax + by.$$

$$\frac{\partial U}{\partial y} = bx$$

$$\frac{\partial U}{\partial z} = -2cz^2.$$

$$\therefore \vec{E} = -(2ax + by)\vec{i} - bx\vec{j} + 2cz^2\vec{k}$$

9.25 $\hookrightarrow W = q_0 (U_C - U_D) = q_0 \int_0^c E dl$
 $= q_0 \int_0^c \frac{q}{4\pi\epsilon_0 r^2} dl = \frac{q_0 q}{6\pi\epsilon_0 l} \quad (\text{EWS})$
 $\omega W_1 = -\int_C^{+\infty} q_0 E dl = -\int_C^{+\infty} q_0 \frac{q}{4\pi\epsilon_0 r^2} dl = -\frac{q_0 q_1}{12\pi\epsilon_0 l}$
 $W_2 = \int_C^{+\infty} q_0 E dl = \int_C^{+\infty} q_0 \frac{q}{4\pi\epsilon_0 r^2} dl = \frac{q_0 q_1}{4\pi\epsilon_0 l}$
 $W_{\text{ES}} = W_1 + W_2 = \frac{q_0 q_1}{6\pi\epsilon_0 l}$

9.29. AB: $E = \int_l^{l+R} \frac{\lambda dl}{4\pi\epsilon_0 r^2} = \frac{\lambda}{4\pi\epsilon_0} \left(\frac{1}{l} - \frac{1}{l+R} \right)$
 $U_{AB} = \int_R^{+\infty} E dl = \frac{\lambda}{4\pi\epsilon_0} \int_R^{+\infty} \left(\frac{1}{l} - \frac{1}{l+R} \right) dl = \frac{\lambda \ln l}{4\pi\epsilon_0}$
 CD: $U_{CD} = U_{AB} = \frac{\lambda \ln l}{4\pi\epsilon_0} \quad (\text{同理}) = \frac{\lambda}{4\pi\epsilon_0}$
 $\widehat{BC}: dE = \frac{\lambda R d\theta}{4\pi\epsilon_0 r^2} \quad U_{\widehat{BC}} = \int_R^{+\infty} \int_0^\pi dE dl = \frac{\lambda R}{4\epsilon_0} \int_R^{+\infty} \frac{1}{l^2} dl$
 $\therefore U_0 = U_{AB} + U_{\widehat{BC}} - U_{CD} = \frac{\lambda}{4\epsilon_0} \left(\frac{2 \ln l}{\pi} + 1 \right)$

9.21. $\omega \lambda = \frac{Q}{2l} \quad dq = \frac{Q}{2l} dh \quad dU = \frac{1}{4\pi\epsilon_0} \cdot dq \cdot \frac{1}{\sqrt{h^2+y^2}}$
 $U = \int_{-l}^l \frac{Q}{8\pi\epsilon_0 l} \cdot \frac{1}{\sqrt{y^2+h^2}} dh = \frac{Q}{4\pi\epsilon_0 l} \ln \frac{\sqrt{y^2+l^2}+l}{y}$
 $E = -\text{grad } U = -\frac{\partial U}{\partial y} \vec{j} = \frac{Q}{4\pi\epsilon_0 l} \frac{y}{\sqrt{y^2+l^2}} \cdot \frac{y^2}{\sqrt{y^2+l^2} - \sqrt{y^2+l^2} - l} \vec{j}$
 $= \frac{Q}{4\pi\epsilon_0 y} \cdot \frac{1}{\sqrt{y^2+l^2}} \vec{j}$
 $\omega \lambda = \frac{Q}{2l} \quad dq = \lambda dr = \frac{Q}{2l} dr$
 $dU = \frac{1}{4\pi\epsilon_0} \frac{dq}{r} = \frac{Q}{8\pi\epsilon_0 l} \cdot \frac{dr}{r} \quad U = \int_{\pi-2l}^{\pi+l} dU$
 $\therefore U = \frac{Q}{8\pi\epsilon_0 l} \ln \frac{\pi+l}{\pi-2l}$
 $E = -\text{grad } U = -\frac{\partial U}{\partial x} \vec{i} = -\frac{Q}{8\pi\epsilon_0 l} \cdot \frac{\pi-2l}{\pi+l} \cdot \frac{(-2l)}{(\pi-2l)^2} \vec{i}$
 $\therefore E = \frac{Q}{4\pi\epsilon_0 (x^2-l^2)} \vec{i}$



9.32 l 为侧面长度 $R_1/\sin\theta \sim R_2/\cos\theta$

$$dq = \sigma \cdot 2\pi l \sin\theta \cdot dl \quad dU = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{l} = \frac{\sigma \sin\theta}{2\epsilon_0} dl$$

$$U = \int dU = \int_{R_1/\sin\theta}^{R_2/\cos\theta} = \frac{\sigma}{2\epsilon_0} (R_2 - R_1)$$

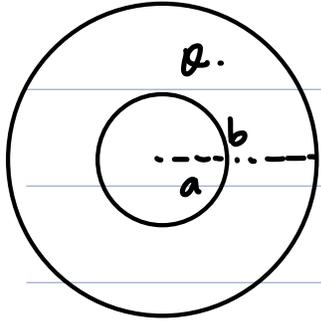
9.33

$$S = \pi(b^2 - a^2) \quad \sigma = \frac{Q}{S} \quad dq = \sigma \cdot 2\pi r \cdot dr \quad (r \in [a, b])$$

$$dU = \frac{1}{4\pi\epsilon_0} \frac{dq}{r} = \frac{\sigma}{2\epsilon_0} dr \quad U = \int dU = \frac{\sigma}{2\epsilon_0} \int_a^b dr$$

$$\therefore U = \frac{\sigma}{2\epsilon_0} (b-a) = \frac{Q \cdot (b-a)}{2S\epsilon_0} = \frac{Q}{2\pi(a+b)\epsilon_0}$$

$$\text{经计算 } U = 9 \times 10^3 \text{ V.}$$



10.1 空腔内壁带电 $-q$. 空腔外壁带电 $+(Q+q)$

$$U = U_{q_1} + U_{内} + U_{外} = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r} - \frac{q}{a} + \frac{Q+q}{b} \right)$$

$$10.4 \quad 1) \quad U = U_{q_1} + U_0 \quad U_{q_1} = \frac{q_1}{4\pi\epsilon_0 l}$$

$$U_0 = \int_S \frac{\sigma dS}{4\pi\epsilon_0 R} = \frac{1}{4\pi\epsilon_0 R} \int_S \sigma dS = 0 \quad (\text{电中性}).$$

$$\therefore U = U_{q_1} + U_0 = \frac{q_1}{4\pi\epsilon_0 l}$$

$$\text{又 } U = \frac{1}{4\pi\epsilon_0 R} \int_S \sigma' dS + \frac{q_1}{4\pi\epsilon_0 l} = 0$$

$$q_1 = \int_S \sigma' dS = -q \frac{R}{l}$$

$$10.6 \quad 1) \quad E = \begin{cases} \frac{q}{4\pi\epsilon_0 R^2} & R_1 < R < R_2 \\ 0 & R_2 < R < R_3 \\ \frac{Q+q_1}{4\pi\epsilon_0 R^2} & R_3 < R \end{cases} \quad (\text{高斯定理}).$$

$$U_2 = \int_{R_2}^{+\infty} \vec{E} \cdot dR = \frac{Q+q_1}{4\pi\epsilon_0 R_3}$$

$$U_1 = U_2 + \int_{R_1}^{R_2} \vec{E} \cdot dR = \frac{Q+q_1}{4\pi\epsilon_0 R_3} + \frac{q_1}{4\pi\epsilon_0} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

12) 由上知 $U_1 = U_2 = \frac{Q+q_2}{4\pi\epsilon_0 R_3}$

13) 设内球电荷量 q_2 。

$$E = \begin{cases} q_2 / 4\pi\epsilon_0 R^2 & R_1 < R < R_2 \\ 0 & R_2 < R < R_3 \\ (Q+q_2) / 4\pi\epsilon_0 R^2 & R_3 < R \end{cases}$$

$$U_1' = \frac{Q+q_2}{4\pi\epsilon_0 R_3} + \frac{q_2}{4\pi\epsilon_0} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = 0$$

$$\therefore q_2 = \frac{Q}{R_3} / \left(\frac{1}{R_2} - \frac{1}{R_1} - \frac{1}{R_3} \right)$$

$$U_2' = \frac{Q+q_2}{4\pi\epsilon_0 R_3} = \frac{Q}{4\pi\epsilon_0 R_3} \left[1 + \frac{1}{R_3} / \left(\frac{1}{R_2} - \frac{1}{R_1} - \frac{1}{R_3} \right) \right]$$

静电平衡的形成

10. (11 12 14 15 18 . 9)

- * 外电场引起导体内自由电子移动, 导体带等量异号感应电荷.
- * 感应电荷激发附加电场, 改变导体内外电场.
- * 外电场与附加电场恰好抵消时, 自由电子停止宏观运动. 达到静电平衡状态.

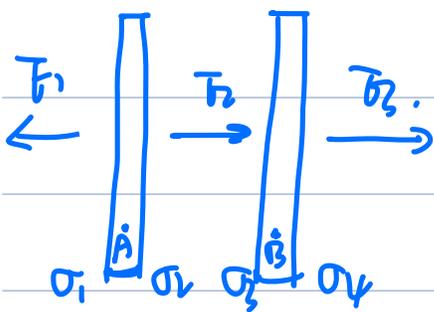
静电平衡的特点.

- ① * 导体内部场强处处为0. 外表面紧靠表面外场强与表面垂直.
* 导体是一个等势体, 导体表面是一个等势面.
- ② * 电荷只分布在导体表面上.
* 电荷正数和负
* 场强与电荷面密度关系. $E = \frac{\sigma}{\epsilon_0}$. ($E \cdot S = \frac{1}{\epsilon_0} \Sigma q$)
* 孤立导体曲率越大处, 电荷面密度越大.

静电屏蔽

- * 外部电场不会影响到空腔内部, 只影响外表面电荷分布.
- * 外表面未接地, 空腔内电荷影响导体外电场.
- * 若接地(电势为0), 则内外电场互不干扰.

eg1. 面积 S 的金属薄板 A. B. A 带电 Q . B 不带电. 两板间距远小于其线度, 二者平行放置. 试求两板表面电荷面密度, 以及其周围场强分布.



$$\sigma_1 + \sigma_2 = \frac{Q}{S}, \quad \sigma_3 + \sigma_4 = 0.$$

$$F_A: \frac{\sigma_1}{2\epsilon_0} - \frac{\sigma_2}{2\epsilon_0} - \frac{\sigma_3}{2\epsilon_0} - \frac{\sigma_4}{2\epsilon_0} = 0.$$

$$F_B: \frac{\sigma_1}{2\epsilon_0} + \frac{\sigma_2}{2\epsilon_0} + \frac{\sigma_3}{2\epsilon_0} - \frac{\sigma_4}{2\epsilon_0} = 0.$$

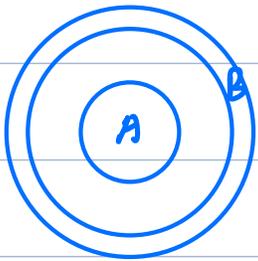
解得 $\sigma_1 = \sigma_2 = \sigma_4 = \frac{Q}{2S}, \quad \sigma_3 = -\frac{Q}{2S}.$

$$F_1 = -\frac{\sigma_1}{2\epsilon_0} - \frac{\sigma_2}{2\epsilon_0} - \frac{\sigma_3}{2\epsilon_0} - \frac{\sigma_4}{2\epsilon_0} = -\frac{Q}{2\epsilon_0 S}$$

$$F_2 = \frac{\sigma_1}{2\epsilon_0} + \frac{\sigma_2}{2\epsilon_0} - \frac{\sigma_3}{2\epsilon_0} - \frac{\sigma_4}{2\epsilon_0} = \frac{Q}{2\epsilon_0 S}.$$

$$F_3 = \frac{\sigma_1}{2\epsilon_0} + \frac{\sigma_2}{2\epsilon_0} + \frac{\sigma_3}{2\epsilon_0} + \frac{\sigma_4}{2\epsilon_0} = \frac{Q}{2\epsilon_0 S}.$$

eg2. 导体球A半径 R_1 . 带电 q . 球外有一内外半径分别为 R_2, R_3 的同心球壳B. 带电 Q .



1) 求球A和球壳B的电势.

2) 用细导线连接A、B. 再求其电势.

3) 若未连接时将球壳接地, 此时电势为多少.

4) 球壳接地再拆除. 再将球接地, 求二者电势.

1) A表面带电 q . B内表面带电 $-q$. 外表面 $Q+q$.

$$\therefore E = \begin{cases} \frac{Q+q}{4\pi\epsilon_0 r^2} & r > R_3. \\ 0 & 0 < r < R_1, R_2 < r < R_3. \\ \frac{q}{4\pi\epsilon_0 r^2} & R_1 < r < R_2. \end{cases}$$

$$U_B = \int_{R_3}^{\infty} \frac{Q+q}{4\pi\epsilon_0 r^2} dr = \frac{Q+q}{4\pi\epsilon_0 R_3}.$$

$$U_A = U_B + \int_{R_1}^{R_2} \frac{q}{4\pi\epsilon_0 r^2} dr = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) + \frac{Q+q}{4\pi\epsilon_0 R_3}.$$

2) $U_A = U_B = \frac{Q+q}{4\pi\epsilon_0 R_3}.$

3) B外表面电势为0. A的电量不变.

$$U_B = 0. \quad U_A = U_B + \int_{R_1}^{R_2} \frac{q}{4\pi\epsilon_0 r^2} dr = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{R_1} - \frac{1}{R_2} \right).$$

4) 设A带电荷 q_x . B带表面电荷 $-q_x$. B外表面电荷 $q_x - q$.

$$U_A = \frac{q_x - q}{4\pi\epsilon_0 R_3} + \frac{q_x}{4\pi\epsilon_0} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = 0.$$

$$\therefore q_x = \frac{q}{R_3} / \left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{1}{R_3} \right)$$

$$U_B = \frac{q_x - q}{4\pi\epsilon_0 R_3} = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{R_3} / \left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{1}{R_3} \right) - 1 \right]$$

电容器的电容

两个等量异号导体组成的电容器, 之间电势差为 $U_A - U_B$.

则电容定义为 $C = \frac{Q}{U_A - U_B}$ (F, nF, pF)

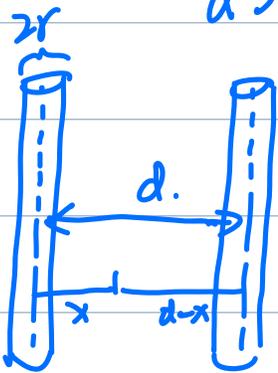
电容器的串并联.

$$\text{串联 } \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n}$$

$$\text{并联 } C = C_1 + C_2 + C_3 + \dots + C_n$$

eg3. 半径 r 的两根 "无限长" 输电线 A, B 轴间距为 d .

$d \gg r$. 求两输电线单位长度的电容.



设输电线中电荷线密度分别为 $\pm\lambda$.

$$x \text{ 处电场 } E = \frac{\lambda}{2\pi\epsilon_0} \left(\frac{1}{x} + \frac{1}{d-x} \right).$$

$$U = \int_r^{d-r} \frac{\lambda}{2\pi\epsilon_0} \left(\frac{1}{x} + \frac{1}{d-x} \right) dx.$$

$$= \frac{\lambda}{2\pi\epsilon_0} \cdot 2 \cdot \ln \frac{d-r}{r} \approx \frac{\lambda}{\pi\epsilon_0} \ln \frac{d}{r}.$$

$$C = \frac{Q}{U} = \frac{\lambda}{U} = \pi\epsilon_0 / \ln \frac{d}{r}.$$

n种典型的电容器.

1) 平行板电容器.

$$E = \frac{\sigma}{\epsilon_0} \quad U_A - U_B = \int_A^B \vec{E} \cdot d\vec{l} = \frac{\sigma d}{\epsilon_0} = \frac{q d}{\epsilon_0 S}$$

$$C = \frac{q}{U_A - U_B} = \frac{\epsilon_0 S}{d} \quad \text{正比于正对面积 反比于距离}$$

2) 同轴圆柱形电容器.

长 l . 半径 R_A, R_B . $l \gg (R_A - R_B)$.

$$\lambda = \frac{q}{l} \quad E = \frac{\lambda}{2\pi\epsilon_0 r} \quad R_A < r < R_B$$

$$U_A - U_B = \int_A^B \vec{E} \cdot d\vec{l} = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{R_B}{R_A}$$

$$C = \frac{q}{U_A - U_B} = \frac{2\pi\epsilon_0 l}{\ln R_B / R_A}$$

3) 同心球电容器.

$$E = \frac{q}{4\pi\epsilon_0 r^2} \quad U_A - U_B = \int_A^B \vec{E} \cdot d\vec{l} = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{R_A} - \frac{1}{R_B} \right)$$

$$C = \frac{q}{U_A - U_B} = \frac{4\pi\epsilon_0 R_A R_B}{R_B - R_A}$$

电场中的电介质参数

① 电介质在电场中的表现

- * 外电场加在电介质上, 同样在介质表面产生电荷 (极化)
- * 介质内部电场不会与外电场抵消, 内场强不为 0.

② 极化强度.

* 某点对应体积微元中所有分子电矩矢量和为 P .

* 外加电场不太大时有线性关系 $P = \epsilon_0 \chi_e E = \epsilon_0 (\epsilon_r - 1) E$
(电极化率 χ_e , 相对介电常数 ϵ_r).

③ 极化电荷面密度.

* 均匀电介质极化时, 电介质表面上某点处的极化电荷面密度 σ' 等于极化强度在该点的法向分量

$$\sigma' = P \cdot e_n.$$

电介质下的电场定理

① 电位移 D .

* 为? 不考虑极化电场和附加电场. 引入电位移 D .

$$D = \epsilon_0 E + P.$$

由 $P = \epsilon_0 \chi_e E$: $D = (1 + \chi_e) \epsilon_0 E = \epsilon_r \epsilon_0 E = \epsilon E$.

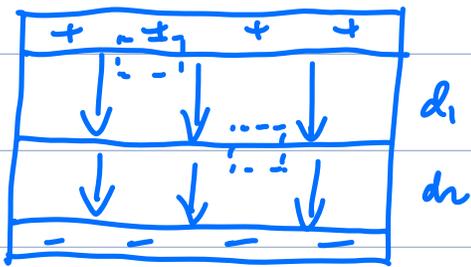
(ϵ 为介电常数).

② 电介质中的高斯定理.

* 通过电场中任意闭合曲面的位移电通量等于该闭合曲面所包围自由电荷的代数和.

$$\oint_S D \cdot dS = \sum q_f$$

eg 4 平行板电容器两极板面积为 S . 间距为 d . 在两板间平行放置两块厚度分别为 d_1, d_2 的介质板. 相对介电常数分别为 ϵ_{r1} 和 ϵ_{r2} . 设电容器充电后两板分别带有 $\pm q$ 的电荷. 求 (1) 电容器的电容; (2) 两介质交界面上极化电荷面密度.



∴

① 求 D : $D_1 S = q, D_2 S = q$

∴ $D_1 = D_2 = q/S$.

② 求 E : $\oint_S D dS = \sum q_r$

∴ $E_1 = \frac{q}{\epsilon_{r1} \epsilon_0 S}, E_2 = \frac{q}{\epsilon_{r2} \epsilon_0 S}$.

③ 求 U : $U = \int_0^{d_1} E_1 dx + \int_{d_1}^{d_1+d_2} E_2 dx = \frac{q d_1}{\epsilon_{r1} \epsilon_0 S} + \frac{q d_2}{\epsilon_{r2} \epsilon_0 S}$.

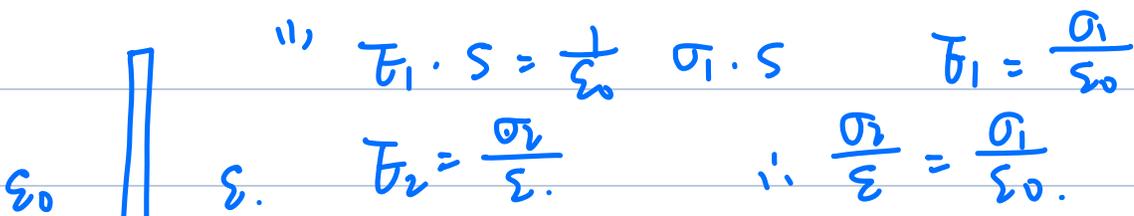
④ 求 C : $C = \frac{q}{U} = S / (\frac{d_1}{\epsilon_{r1}} + \frac{d_2}{\epsilon_{r2}})$.

∴ 两电荷交界面上极化电荷面密度总量等于两介质界面上电荷面密度的代数和 $\vec{P} = \epsilon_0 (\epsilon_r - 1) \vec{E}$ 均向下.

$\sigma' = \sigma_1' + \sigma_2' = \vec{P}_1 \cdot \vec{n}_1 + \vec{P}_2 \cdot \vec{n}_2 = P_1 - P_2 = \frac{\epsilon_{r1} - \epsilon_{r2}}{\epsilon_{r1} \epsilon_{r2}} \frac{q}{S}$.

eg 5. 真空中一无限大带电导体板两侧面上电荷面密度均为 σ .

现在导体板右侧充满介电常数为 ϵ 的均匀电介质. 求 (1) 导体板左、右侧面上自由电荷面密度 σ_1, σ_2 . 以及电介质表面极化面密度 σ' (2) 导体板左、右场强大小及方向.



∴ $E_1 \cdot S = \frac{1}{\epsilon_0} \sigma_1 \cdot S, E_1 = \frac{\sigma_1}{\epsilon_0}$

$E_2 = \frac{\sigma_2}{\epsilon}, \therefore \frac{\sigma_2}{\epsilon} = \frac{\sigma_1}{\epsilon_0}$.

由电荷守恒知 $\sigma_1 + \sigma_2 = 2\sigma$.

$$\text{解得 } \sigma_1 = 2\sigma \frac{\epsilon_0}{\epsilon_0 + \epsilon} \quad \sigma_2 = 2\sigma \frac{\epsilon}{\epsilon_0 + \epsilon}$$

$$\sigma' = \sigma_1 - \sigma_2 = 2\sigma \frac{\epsilon_0 - \epsilon}{\epsilon_0 + \epsilon}$$

$$\therefore E_1 = E_2 = \frac{2\sigma}{\epsilon_0 + \epsilon} \quad E_1 \text{ 向左, } E_2 \text{ 向右}$$

eg 6. 一平行板电容器. 两板间距为 d . 带电量分别为 $\pm q$. 板间电势为 U . 忽略边缘效应. 断开电源, 两板间插入一厚度为 t ($t < d$) 的金属板. 求板间电势差和电容值.

$$E = \frac{U}{d} = \frac{U'}{d-t} \quad U' = \frac{d-t}{d} U \quad C' = \frac{q}{U'} = \frac{d}{d-t} \cdot \frac{q}{U}$$

eg 7. 一球形电容器. 内外球面半径分别为 $R_1 = 2 \text{ cm}$, $R_2 = 4 \text{ cm}$. 两球间充满击穿电场强度为 100 kV/m 的电介质. 则该电容器能耐受的最大电压为. _____.

$$E = \frac{q}{4\pi\epsilon_0 r^2} \quad (R_1 < r < R_2) \quad \therefore E_{\max} = \frac{q}{4\pi\epsilon_0 R_1^2} \leq E_{\text{击穿}}$$

$$\therefore q_{\max} = 4\pi\epsilon_0 R_1^2 \cdot E_{\text{击穿}}$$

$$U_{\max} = \int_{R_1}^{R_2} \frac{q_{\max}}{4\pi\epsilon_0 r^2} dr = R_1^2 E_{\text{击穿}} \cdot \int_{R_1}^{R_2} \frac{1}{r^2} dr$$

$$= R_1^2 \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \cdot E_{\text{击穿}} = 1000 \text{ V}$$

静电场中能量.

$$W = \frac{1}{2} \sum q_i U_i \quad \text{点电荷}$$

(W 系统总能量. $U_i = q_i$ 处所有 q_i 以外的电荷产生的电势.)

若电荷连续分布. 改写成微元电荷积分即可.

$$W = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} Q (U_A - U_B) = \frac{1}{2} C (U_A - U_B)^2 \quad \text{电容器}$$

$$\text{电能密度 } w_e = \frac{W}{V} = \frac{1}{2} \epsilon F^2 = \frac{1}{2} D E.$$

$$\text{非均匀电场 } W = \int_V w_e dV.$$

eg 8. 半径为 a 的长直导线. 外有共轴导体圆筒. 内半径 b .

其间充满相对介电常数 ϵ_r 的均匀电介质. 单位长度导线均匀带正 λ . 圆筒 $-\lambda$. 求沿轴线单位长度电场能量.

$$D = \frac{\lambda}{2\pi r} \quad E = \frac{D}{\epsilon_r \epsilon_0} = \frac{\lambda}{2\pi \epsilon_r \epsilon_0 r}$$

$$w_e = \frac{1}{2} D E = \frac{\lambda^2}{8\pi^2 \epsilon_r \epsilon_0 r^2}.$$

$$\text{柱坐标系积分: } W = \int_V w_e dV \quad dV = 2\pi r l dr$$

$$\begin{aligned} \therefore W &= \int_a^b \frac{\lambda^2}{8\pi^2 \epsilon_r \epsilon_0} \frac{1}{r^2} \cdot r dr \int_0^{2\pi} d\phi \int_0^l dz \\ &= \frac{\lambda^2 l}{4\pi \epsilon_r \epsilon_0} \ln \frac{b}{a} \Rightarrow \frac{W}{l} = \frac{\lambda^2}{4\pi \epsilon_r \epsilon_0} \ln \frac{b}{a}. \end{aligned}$$

eg 9. 平行板电容器极板面积 $2 \times 10^{-2} \text{ m}^2$. 板间距 $1 \times 10^{-3} \text{ m}$.

电容器内有一介质板 $\epsilon_r = 5$. 充满两板间全部空间

电容器与 300 V 电源相连. 充电后切断电源. 再抽出介质板.

求: (1) 抽出过程中外力做功.

(2) 抽出后两板间相互作用力.

$$\begin{aligned} \text{1) } C_0 &= \frac{\epsilon_0 \epsilon_r S}{d} & W_0 &= \frac{Q^2}{2C_0} \\ C_1 &= \frac{\epsilon_0 S}{d} & W_1 &= \frac{Q^2}{2C_1}. \end{aligned}$$

$$A = W_1 - W_0 = \frac{1}{2} Q^2 \left(\frac{1}{C_1} - \frac{1}{C_0} \right).$$

$$= \frac{1}{2} (C_0 U)^2 \left(\frac{1}{C_1} - \frac{1}{C_0} \right) = 1.593 \times 10^{-4} \text{ J}.$$

$$0) \quad W = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \frac{Q^2 x}{\epsilon_0 S}.$$

能量来源于电场力做功 $dA = F dx$. 同时 $dA = -dW$

$$\therefore F = \frac{dA}{dx} = - \frac{dW}{dx} = - \frac{Q^2}{2\epsilon_0 S} = -0.199 \text{ N}.$$

电流强度 I : $I = \frac{dq}{dt}$ 单位时间通过截面的电荷量.

电流密度 j : 描述在导体内某一点的电流情况

$$I = \int_S j ds. \quad j = \frac{dI}{ds}.$$

自由电子除无规则热运动外, 还会逆着电场定向运动(漂移)

漂移平均速度 v_d 与电流密度关系 $\vec{j} = -nev_d$.

n 为自由电子数密度. e 为基本电荷 ($1.6 \times 10^{-19} C$).

稳恒电流. 导体中各点电流密度矢量 j 或单位时间通过.

导体任一截面的电流强度不随时间变化.

$$\oint_S j ds = -\frac{dq}{dt} = 0.$$

eg1. 直径 2 mm 的铜丝中有 1 A 的稳恒电流.

则导线中电子的漂移速度为 ____ . 铜中电子浓度

$$j = \frac{I}{S} = \frac{4I}{\pi d^2}. \quad 8.5 \times 10^{28} / m^3.$$

$$v_d = \frac{j}{ne} = \frac{4I}{ne\pi d^2} = 2.34 \times 10^{-5} m/s.$$

电势差产生电流, 稳恒条件下一定时间后导体的电流强度

与导体两端电势差成正比 $I = \frac{U}{R}$. **欧姆定律.**

一定温度下导体的电阻决定于其本身性质和几何形状.

$$R = \rho \frac{L}{S}. \quad (\rho: \text{电阻率}).$$

电阻率的倒数 γ ($\gamma = \frac{1}{\rho}$) 称为电导率.

电阻率与温度间大致有如下线性关系 $\rho_t = \rho_0(1 + \alpha \cdot t)$.

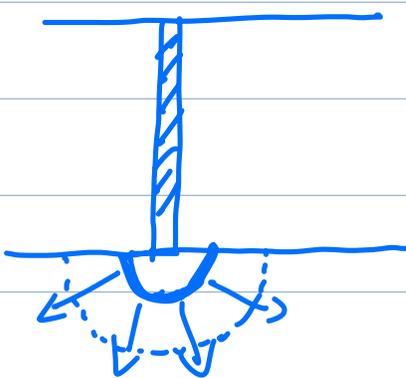
ρ_0 为 0°C 时电阻率. α 为电阻温度系数.

欧姆定律的微分形式. $\Delta U = E \Delta l$. $\Delta I = j \Delta S$.

$$E \Delta l = j \Delta S \Delta R.$$

$$\text{即 } j = \frac{\Delta I}{\Delta S} = \frac{E \Delta l}{\Delta R \Delta S} = \frac{1}{\rho} E = \gamma E. \quad \text{方向一致 } \vec{j} = \gamma \vec{E}$$

eg2



输电钢架埋入地下部分可近似当作半个球体.

大地电导率视作均匀 $\gamma = 1 \times 10^{-2} \text{ S/m}$.

500A 的短路电流从钢架流入大地.

有人以 0.6m 步幅向钢架中心走去.

求其前脚距中心 2m 时的跨步电压.

$$E = \frac{j}{\gamma} = \frac{1}{\gamma} \cdot \frac{I}{2\pi r^2} \quad (2\pi r^2 \text{ 为半球面积}).$$

$$U = \int_{r_1}^{r_2} E dr = \frac{I}{2\pi\gamma} \left(-\frac{1}{r}\right) \Big|_{r_1}^{r_2} = \frac{I}{2\pi\gamma} \left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

$$= 918.67 \text{ V}.$$

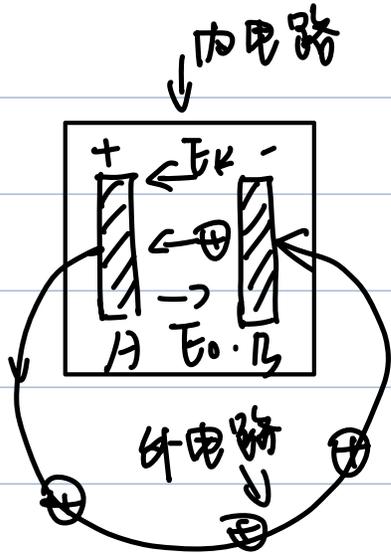
焦耳定理的微分形式.

$$Q = I^2 R t. \quad P = I^2 R.$$

$$\Delta P = (\Delta I)^2 \Delta R = (j \Delta S)^2 \left(\rho \frac{\Delta l}{\Delta S}\right) = \rho j^2 \Delta V = \gamma E^2 \Delta V.$$

并称单位体积内的热功率密度以 w 表示.

$$w = \frac{\Delta P}{\Delta V} = \gamma E^2.$$



导体A带正电. 电势为 U_A . 导体B带负电. 电势为 U_B

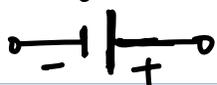
并设 $U_A > U_B$. 用导线连接A. B.

为形成稳恒电流需有提供非静电力外的电源.

使 \oplus 从负极运动到正极.

电源中的非静电力将正电荷 q_+ 经内电路从负极到正极的过程. 要克服的静电力做功为 A_k . 使正电荷的电势能增加

电动势 $\mathcal{E} = \frac{A_k}{q_+}$ (正方向为从负极指向正极).



可设想非静电力由非静电场 E_k 产生

q_+ 从负极到正极过程中 $A_k = q_+ \int_-^+ E_k \cdot dl$.

$$\therefore \mathcal{E} = \frac{A_k}{q_+} = \int_-^+ E_k \cdot dl$$

对非静电力分布在各处的情况 $\mathcal{E} = \oint E_k \cdot dl$.

电源电动势数值上等于单位正电荷绕闭合回路一周时非静电力做的功.

电源的路端电压 $U_+ - U_- = \int_+^- E_0 \cdot dl$.

电源内部既有静电场 \vec{E}_0 . 又有非静电场 \vec{E}_k . 合场 $\vec{E} = \vec{E}_0 + \vec{E}_k$

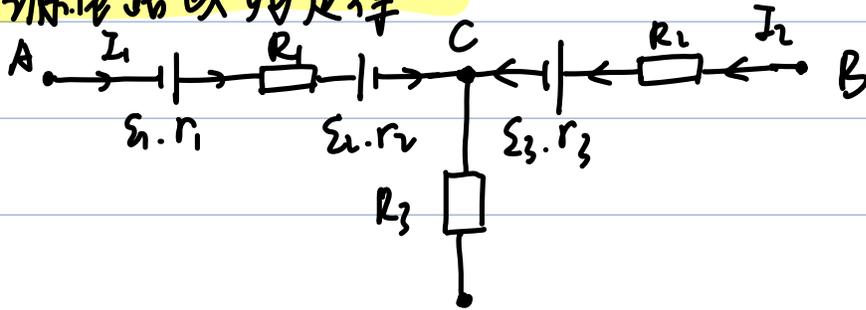
$$\vec{j} = \gamma (\vec{E}_0 + \vec{E}_k). \quad \vec{E}_0 = \frac{1}{\gamma} \vec{j} - \vec{E}_k$$

$$U_+ - U_- = - \int_+^- \vec{E}_k \cdot dl + \frac{1}{\gamma} \int_+^- \vec{j} \cdot dl$$

$$= \int_-^+ \vec{E}_k \cdot dl + \rho \int_+^- \frac{I}{S} \cdot dl$$

$$= \Sigma - I \cdot r \quad (r \text{ 为电源内电阻}).$$

含源电路欧姆定律



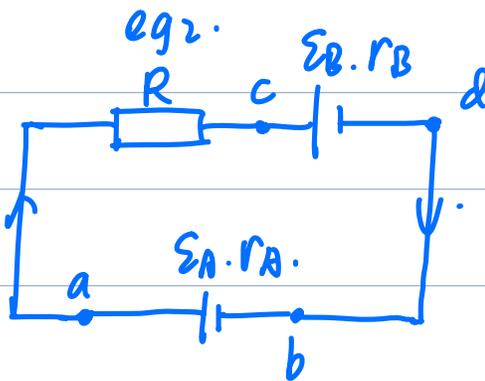
考虑 U_C . $U_C = U_A - I_1 r_1 - I_1 R_1 - I_1 r_2 + \Sigma_1 - \Sigma_2$.

$$U_C = U_B - I_2 R_2 - I_2 r_3 - \Sigma_3$$

$$\therefore U_B - U_A = (\Sigma_1 - \Sigma_2 + \Sigma_3) - I_1 (r_1 + R_1 + r_2) + I_2 (r_3 + R_2)$$

$$U_B - U_A = \Sigma \Sigma - \Sigma IR$$

若将 A、B 相连. $U_B = U_A \quad \Sigma \Sigma - \Sigma IR = 0 \quad I = \frac{\Sigma \Sigma}{\Sigma R}$



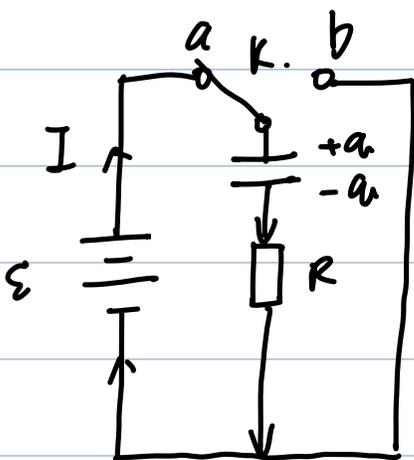
$$\Sigma_A = 24V, \quad r_A = 2\Omega, \quad \Sigma_B = 12V, \quad r_B = 1\Omega$$

$$R = 3\Omega, \quad \text{求电流及 } U_{ab}$$

$$\Sigma \Sigma = 12V, \quad \Sigma R = 6\Omega, \quad I = \frac{\Sigma \Sigma}{\Sigma R} = 2A$$

$$U_a = U_b + \Sigma_A - I r_A = U_b + 20V$$

$$U_{ab} = U_b - U_a = -20V$$



K 接 a 时某时刻 t, 电路中电流为 I.
电容器极板上的电量为 q, 电势差 U.

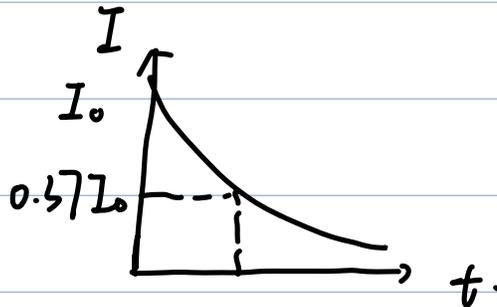
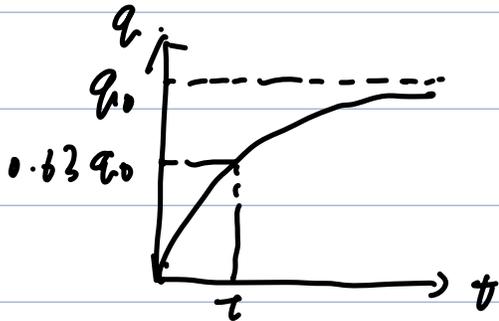
$$\Sigma - U - IR = 0, \quad U = \frac{q}{C}, \quad I = \frac{dq}{dt}$$

$$R \frac{dq}{dt} + \frac{q}{C} - \Sigma = 0$$

若初始条件 $q_0=0$. $t=0$. 积分得

$$q = C\varepsilon (1 - e^{-t/RC}) = q_0 (1 - e^{-t/RC}) .$$

$$I = \frac{dq}{dt} = \frac{\varepsilon}{R} e^{-t/RC} = I_0 e^{-t/RC} .$$



电流和电量按指数随时间变化. 速率取决于 RC 乘积. 时间常数.

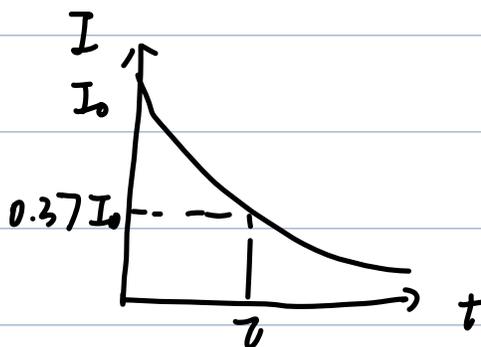
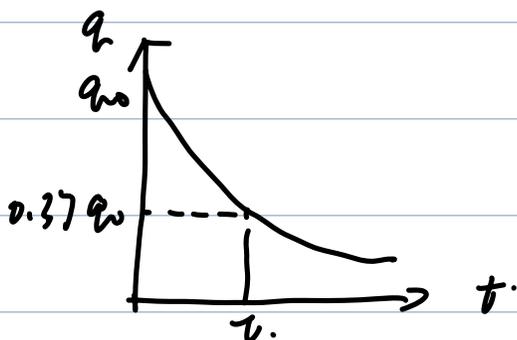
$$\tau = RC \quad q = q_0 (1 - e^{-1}) = 0.63 q_0 .$$

$$I = I_0 e^{-1} = 0.37 I_0 .$$

$$k \text{ 接 } b \text{ 时. } \frac{q}{C} - IR = 0 \Rightarrow \frac{q}{C} + R \frac{dq}{dt} = 0 .$$

$$t=0 \text{ 时 } q = q_0$$

$$q = q_0 e^{-t/RC} . \quad I = - \frac{dq}{dt} = I_0 e^{-t/RC} .$$

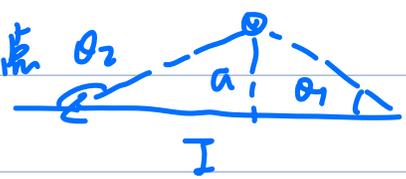


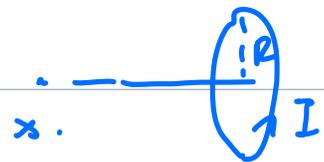
$\tau = RC$ 也是放电快慢的标志.

毕奥-萨伐尔. 长 dl 的电流元 I . 在 P 处产生的磁感应强度.

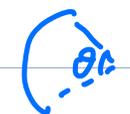
$$dB = \frac{\mu_0}{4\pi} \frac{I dl \times r}{r^3}$$

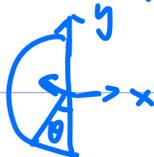
某点的磁感应强度满足叠加原理.

eg. 长直导线外一点  $B = \frac{\mu_0 I}{4\pi a} (\cos\theta_1 - \cos\theta_2)$

圆线圈外一点  $B = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$

无限长直导线外一点  $B = \frac{\mu_0 I}{2\pi a}$

圆环形电流圆心处,  $B = \frac{\mu_0 I}{2R} \cdot \frac{1}{2\pi}$

eg 半径 R 的半圆导线上有均匀的电流 I . 求轴线上一点 P

 电流线密度 $j = \frac{I}{\pi R}$. $dI = j dl = j R d\theta$.

由对称性得 $B_x = 0$.

$$dB_y = \frac{\mu_0 \cdot dI}{2\pi R} \sin\theta = \frac{\mu_0 j R d\theta}{2\pi R} \sin\theta = \frac{\mu_0 I}{2\pi^2 R} \sin\theta d\theta$$

$$B_y = \int dB_y = \frac{\mu_0 I}{2\pi^2 R} \int_0^\pi \sin\theta d\theta = \frac{\mu_0 I}{\pi^2 R}$$

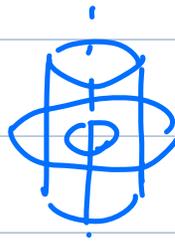
单个电荷 q 以速度 v 运动产生磁感应强度 $B = \frac{\mu_0}{4\pi} \frac{qv \times r}{r^3}$.

安培环路定理: 稳恒磁场中, B 沿任何方向回路线积分为等于
 闭合回路包含电流 $\sum I$ 数和的 μ_0 倍.

$$\oint B \cdot dl = \mu_0 \sum I_{\text{内}}$$

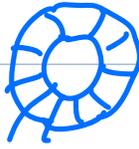
eg

无限长载流圆柱



$$B = \begin{cases} \frac{\mu_0 I}{2\pi r} \\ \frac{\mu_0 I r}{2\pi R^2} \end{cases}$$

载流螺线环



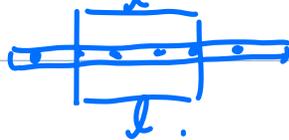
$$B = \frac{\mu_0 N I}{2\pi R} = \mu_0 n I$$

载流螺线管



$$B = \frac{\mu_0 n I}{1} = \mu_0 n I$$

无限大载流平面



$$B = \frac{\mu_0 I \cdot j}{2d} = \frac{\mu_0}{2} j$$

磁通量 $\Phi_m = \int_S B \cdot dS$

磁场高斯定理 $\oint_S B \cdot dS = 0$. 用于通过简单曲面中复杂曲面磁通量

磁矩 $P_m = N I S e_m$

$$dq = \sigma ds = 2\pi \sigma r dr$$

eg.



半径 R 圆盘. 表面均匀分布 +q 电荷. 以 ω 速度旋转.

$$\sigma = \frac{q}{\pi R^2}, \quad dq = \sigma r dr = \frac{q r}{\pi R^2} dr, \quad d = \frac{2\pi}{\omega}$$

$$dI = \frac{dq}{d} = \frac{q \omega r}{2\pi R^2} dr$$

$$P_m = \int S dI = \int_0^R \pi r^2 \frac{q \omega r}{2\pi R^2} dr = \frac{q \omega}{2R^2} \int_0^R r^3 dr = \frac{1}{2} q \omega R^2$$

由右手螺旋定则知方向竖直向上.

安培力是洛伦兹力的宏观表现. 对 B 中电流元 $I dl$.

$$dF = I dl \times B$$

$$F = \int_L I dl \times B; \quad F = IL \times B \text{ (直导线).}$$

线圈所受力矩 $dM = r \times dF = r \times (I dl \times B)$.

均匀磁场中. $M = P_m \times B$. (磁力矩).

磁力做功 $dA = I d\phi$. / $A = IB \Delta S$.

霍尔效应 均匀磁场中的通电薄板. 电流与磁场垂直. 产生电势差.

$p \Leftrightarrow$ 正电 \Leftrightarrow 空穴. $n \Leftrightarrow$ 负电 \Leftrightarrow 电子

正电: $I \times B$ 与 B 反向. 负电: $I \times B$ 与 B 同向.

霍尔电压 $U_H = R_H \frac{IB}{d}$ 霍尔系数 $R_H = \frac{1}{nq}$.

d 为 B 方向厚度. n 为载流子浓度.

磁场中磁感应强度 B . 原磁感应强度 B_0 . 附加磁场磁感应强度 B' .

$$B = B_0 + B'$$

① 顺磁质 B' 与 B_0 同向 $\vec{B} > \vec{B}_0$

② 抗磁质 B' 与 B_0 反向 $\vec{B} < \vec{B}_0$

③ 铁磁质 附加场远强于原场. 同向 $\vec{B} \gg \vec{B}_0$.

磁感应强度 B . 磁化强度 M . 磁场强度 H $\mu_0 \mu_r = \mu$.

$$T \quad A/m \quad A/m$$

$M = j_m$. 总磁化电流强度 = $\oint_L j_m dl$.

有磁介质时的安培环路定理 $\oint_L H dl = \sum I$

$$H = \frac{B}{\mu_0} - M; B = \mu_0 (H + M); M = \chi_m H; B = \mu_0 \mu_r H = \mu H.$$

铁磁质 $B-H$ 曲线. ① μ 不是常数. ② B 存在饱和值.

减小 H . B 的减小呈现滞后性. $H=0$ $B \neq 0$. 剩磁感应强度 B_r .

$H = -H_c$ $B = 0$. H_c 称为矫顽力.

$$dI = \frac{dq}{r} = \sigma dS / \frac{2\pi}{w} = \sigma 2\pi (R \sin\theta) R d\theta \cdot \frac{w}{2\pi}$$

$$dI = \sigma_0 w R^2 \sin\theta \cos\theta d\theta$$

$$dB = \frac{\mu_0 dI (R \sin\theta)^2}{2(R^2 \sin^2\theta + R^2 \cos^2\theta)^{3/2}} = \frac{\mu_0 \sigma_0 w R^4 \sin^3\theta \cos\theta d\theta}{2R^3}$$

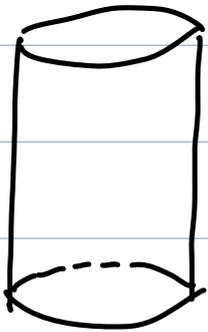
$$= \frac{1}{2} \mu_0 \sigma_0 w R \cdot \sin^2\theta d(\sin\theta).$$

$$B = \frac{1}{2} \mu_0 \sigma_0 w R \int_0^{\frac{\pi}{2}} \sin^2\theta d\sin\theta = \frac{1}{8} \mu_0 \sigma_0 w R.$$

$$d\vec{p}_m = \pi r^2 dI = \pi (R \sin \theta)^2 dI = \pi \sigma_0 \omega R^4 \sin^3 \theta \cos \theta d\theta.$$

$$\vec{p}_m = \int_0^{\frac{\pi}{2}} \pi \sigma_0 \omega R^4 \sin^3 \theta \cos \theta d\theta = \frac{1}{4} \pi \sigma_0 \omega R^4.$$

$$\vec{m} = \vec{p}_m \times \vec{B} = |\vec{p}_m| |\vec{B}| \cdot \sin 90^\circ = \frac{1}{8} \pi \sigma_0 \omega R^4 B$$



$$\textcircled{1} \quad r < R_1 \quad H = 0 \quad B = 0.$$

$$\textcircled{2} \quad R_1 < r < R_2 \quad \oint_{\mathcal{L}} \vec{H} \cdot d\vec{l} = \Sigma I.$$

$$H \cdot 2\pi r = j_1 \cdot 2\pi R_1 \quad B = \mu_0 \mu_r H = \frac{\mu_0 \mu_r j_1 \cdot R_1}{r}.$$

$$\textcircled{3} \quad r > R_2. \quad \oint_{\mathcal{L}} \vec{H} \cdot d\vec{l} = j_1 \cdot 2\pi R_1 + j_2 \cdot 2\pi R_2.$$

$$H = \frac{R_1 j_1 + R_2 j_2}{r} \quad B = \mu_0 H = \frac{\mu_0 (R_1 j_1 + R_2 j_2)}{r}.$$

$$M_1 = \chi_m \cdot H_1 = (\mu_r - 1) j_1 \frac{R_1}{r}. \quad \vec{j}_{m1} = M_1.$$

$$I_{m1} = \vec{j}_{m1} \cdot 2\pi R_1 = 2\pi (\mu_r - 1) j_1 \frac{R_1^2}{r}.$$

$$I_{m2} = 2\pi (\mu_r - 1) \frac{R_2 (R_1 j_1 + R_2 j_2)}{r}.$$

法拉第电磁感应定律.

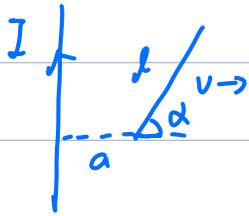
$$\varepsilon_i = - \frac{d\Phi}{dt} \quad \text{全磁通 } \Phi = \Phi_1 + \Phi_2 + \dots + \Phi_n \quad \varepsilon_i = - \frac{d\Phi}{dt}$$

动生电动势

$$d\varepsilon_i = (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l} \quad \text{结果正负号表示与 } d\mathbf{l} \text{ 同/反向.}$$

eg

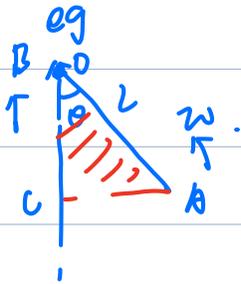
长直导线 I 长 l 的金属棒如图摆放运动. 求此时电动势.



$$B = \frac{\mu_0 I}{2\pi r} \quad dl = \frac{dr}{\cos\alpha}$$

$$d\varepsilon_i = (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l} = Bv \sin\alpha \frac{dr}{\cos\alpha} = \frac{\mu_0 I v \tan\alpha}{2\pi} \frac{dr}{r}$$

$$\varepsilon_i = \int d\varepsilon_i = \int_a^{a+l\cos\alpha} \frac{\mu_0 I v \tan\alpha}{2\pi} \frac{dr}{r} = \frac{\mu_0 I v \tan\alpha}{2\pi} \ln \frac{a+l\cos\alpha}{a}$$



作如图的补充. 回路的电动势为 0.

$$\varepsilon_{OC} = 0 \Rightarrow \varepsilon_{CA} + \varepsilon_{AO} = 0.$$

$$\varepsilon_{CA} = \frac{1}{2} B \omega L^2 \sin\alpha \quad \varepsilon_{OA} = \varepsilon_{CA} = \frac{1}{2} B \omega L^2 \sin\alpha \quad (O \rightarrow A)$$

感生电动势 导体所处磁场变化产生的电动势.

$$\textcircled{1} \text{ 涡旋电场 } \oint_L \mathbf{E}_i \cdot d\mathbf{l} = - \int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$$

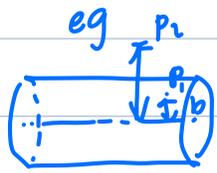
$$\text{圆柱磁场} \begin{cases} 2\pi r \cdot E_i = -\pi r^2 \frac{dB}{dt} & r < R \\ 2\pi r \cdot E_i = -\pi R^2 \frac{dB}{dt} & r > R. \end{cases}$$

$$E_i = \begin{cases} -\frac{r}{2} \frac{dB}{dt} & r < R \\ -\frac{R^2}{2r} \frac{dB}{dt} & r > R. \end{cases}$$

感生电动势为单位电荷绕闭合回路一周电场力做功.

$$\varepsilon_i = \oint_L \mathbf{E}_i \cdot d\mathbf{l}.$$

② 磁通量.



eg 半径 a 无限长带电均匀带电圆筒面. 单位长度电荷 λ .
绕轴匀角加速度 β 转. 求 (1) 距轴 b 的 P_1 的 E_1 .

(2) 如图导体 MN 中电动势.

$$I = \frac{\lambda}{2\pi r} = \frac{\lambda r}{2\pi a} \quad \oint_L B \cdot dl = \mu_0 \Sigma I \quad B \cdot l = \frac{\mu_0 \lambda r l}{2\pi}$$

$$B = \frac{\mu_0 \lambda \beta t}{2\pi} \quad \oint_L \vec{b}_i \cdot d\vec{l} = - \int_S \frac{\partial B}{\partial t} dS$$

$$\vec{b}_i \cdot 2\pi r = - \frac{\mu_0 \lambda \beta}{2\pi} \cdot \pi r^2 \quad b_{i1} = - \frac{\mu_0 \lambda \beta b}{4\pi}$$

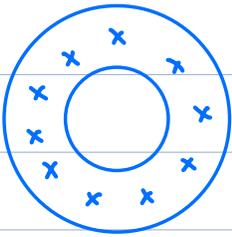
(2) 连接 OM, ON . 回路中 $\Sigma_i = - \frac{d\Phi}{dt} = - S \frac{dB}{dt}$

其中 $S = \frac{2a \arctan \frac{1}{2}}{2\pi} \quad \pi a^2 = \arctan \frac{1}{2} \cdot a^2$.

$$U_{MN} = - \arctan \frac{1}{2} \cdot a^2 \cdot \frac{\mu_0 \lambda \beta}{2\pi}$$

eg 高度 D . 内径 R_1 . 外径 R_2 . 电阻率 ρ 的铜环.

置于垂直环. $B = \frac{t}{r}$ 的磁场中 $r=0$ 点为环中心. 求环上电流.
(磁场限制在环内)



$$\Phi = BS \quad d\Phi = B \cdot dS = 2\pi t dr$$

$$\Phi = 2\pi t \int_{R_1}^{R_2} dr = 2\pi t (R_2 - R_1) \quad \Sigma_i = - \frac{d\Phi}{dt} = -2\pi t (R_2 - R_1)$$

$$R = \rho \frac{L}{S} = \rho \cdot \frac{2\pi r}{D dr}$$

$$dI = \frac{\Sigma_i}{R} = \frac{-2\pi t (R_2 - R_1) D}{\rho \cdot r} dr = \frac{D}{\rho} \left(\frac{R_1}{r} - 1 \right) dr$$

$$I = \int_{R_1}^{R_2} \frac{D}{\rho} \left(\frac{R_1}{r} - 1 \right) dr = \frac{D}{\rho} \left[R_1 \ln \frac{R_2}{R_1} - (R_2 - R_1) \right]$$

eg 两无限长导线平行相距 $2a$. 一根 I_0 一根 $i = I_0 \cos \omega t$

两导线间有一 l . $2b$ 的线圈. $l \parallel$ 电流. $b < a$. 以 v 向右运

动. 线圈在正中时 $\vec{v} = 0$. 求动生. 感生. 感应电动势.

$$\varepsilon_1 = B_1 l v - B_2 l v = \frac{\mu_0 I_0 l v}{2\pi} \left(\frac{1}{a-b} - \frac{1}{a+b} \right) \quad \text{恒定时.}$$



$$\varepsilon_2 = -\frac{d\Phi}{dt} \quad d\Phi = \frac{\mu_0 I l}{2\pi} \frac{dr}{r}$$

$$\Phi = \int_{a-b}^{a+b} \frac{\mu_0 I l}{2\pi} \frac{dr}{r} = \frac{\mu_0 I l}{2\pi} \ln \frac{a+b}{a-b}$$

$$\varepsilon_2 = -\frac{d\Phi}{dt} = -\frac{\mu_0 I_0 l v}{2\pi} \ln \frac{a+b}{a-b} \sin \omega t = \pm \frac{\mu_0 I_0 l v}{2\pi} \ln \frac{a+b}{a-b}$$

$$\varepsilon = \varepsilon_1 + \varepsilon_2 = \frac{\mu_0 I_0 l v}{2\pi} \left(\frac{1}{a-b} - \frac{1}{a+b} \right) \pm \frac{\mu_0 I_0 l v}{2\pi} \ln \frac{a+b}{a-b}$$

① 恒定磁场动线圈：动生。感应。

② 恒定线圈，动磁棒：感生

$$\text{自感: } \Psi = LI \quad \varepsilon_1 = -L \frac{dI}{dt}$$

eg 两根半径 a 的长直导线相距 d $a \ll d$. 求单位长度的自感

$$B = \frac{\mu_0 I}{2\pi r} + \frac{\mu_0 I}{2\pi(d-r)}$$

$$\Phi = \int_a^{d-a} B \cdot l \cdot dr = \frac{\mu_0 I l}{2\pi} \int_a^{d-a} \left(\frac{1}{r} + \frac{1}{d-r} \right) dr$$

$$\therefore \Phi = \frac{\mu_0 I l}{\pi} \ln \frac{d-a}{a} \quad L = \frac{\Phi}{I} = \frac{\mu_0 l}{\pi} \ln \frac{d-a}{a}$$

$$\text{互感: } M_{12} = M_{21} = M. \quad \Psi_{12} = M I_1 \quad \Psi_{21} = M I_2$$

eg 一矩形截面螺线环，由细导线密绕而成，内半径 R_1 外半径 R_2 ，

高为 b ，共 N 匝，环轴线上有一无限长直导线，环内电流 $I = I_0 \cos \omega t$

求无限长直导线中的感应电动势。

$$B = \frac{\mu_0 I}{2\pi r} \quad d\Psi = N d\Phi = N \frac{\mu_0 I b}{2\pi} \frac{dr}{r}$$

$$\Psi = \int d\Psi = \frac{N \mu_0 I b}{2\pi} \int_{R_1}^{R_2} \frac{dr}{r} = \frac{N \mu_0 I b}{2\pi} \ln \frac{R_2}{R_1}$$

$$M = \frac{\Psi}{I} = \frac{N \mu_0 b}{2\pi} \ln \frac{R_2}{R_1}$$

$$\mathcal{E} = -M \frac{dI}{dt} = -\frac{NM\mu_0 b}{2\pi} \ln \frac{R_2}{R_1} \cdot I_0 \omega \sin \omega t$$

自感磁能. 自感 L 的线圈建立稳定电流 I_0 . 磁能 $W_m = \frac{1}{2} L I_0^2$.

磁能密度. $w_m = \frac{1}{2} B \cdot H = \frac{1}{2} \frac{B^2}{\mu}$ 积分得 V 内磁能.

为在非稳恒情况下使用安培环路定理, 引入位移电流.

位移电流密度, $\vec{j}_d = \frac{dD}{dt}$ 位移电流 $I_d = \frac{d\Phi_D}{dt}$

eg 平行板电容器 $S, d, U = U_0 \sin \omega t$. 求 j_d, I_d .

$$j_d = \frac{dD}{dt} = \epsilon_0 \frac{dE}{dt} = \frac{\epsilon_0}{d} \frac{dU}{dt} = \frac{\epsilon_0 U_0 \omega}{d} \cos \omega t.$$

$$I_d = \oint_S j_d \cdot dS = j_d \cdot S = \frac{\epsilon_0 U_0 \omega S}{d} \cos \omega t.$$

麦克斯韦方程组.

$$\oint_S D \cdot dS = \sum q_r$$

$$\oint_L E \cdot dl = - \frac{d\Phi}{dt} = - \int_S \frac{\partial B}{\partial t} \cdot dS.$$

$$\oint_S B \cdot dS = 0$$

$$\oint_L H \cdot dl = \sum I + \frac{d\Phi_D}{dt} = \int_S \vec{j} \cdot dS + \int_S \frac{\partial D}{\partial t} \cdot dS$$

电磁波性质

电磁波是横波. 传播电场 E 和磁场 H . 方向 $E \times H$. 速度 $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$.

能流密度.

S 表示能流密度. 单位时间通过垂直传播方向单位面积的能量.

矢量形式: $S = E \times H$. 坡印廷矢量.

光程：光在介质中传播的路程 \times 介质折射率。

$$\delta = n \cdot r$$

半波损失：光从折射率小介质入射折射率大介质，并被反射回来时，光程会减少 $\frac{\lambda}{2}$ 。 \hookrightarrow 波密。

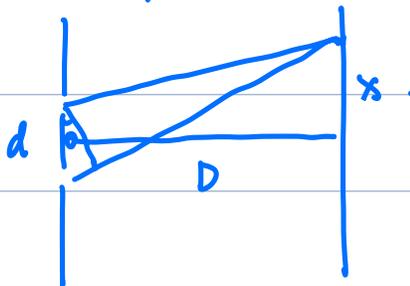
波密介质： $\Delta\varphi = (\varphi_2 - \varphi_1) + \frac{2\pi}{\lambda} \delta$ δ 光程差。

$$\Delta\varphi = \begin{cases} 2k\pi & \text{加强 } k \text{ 级明纹.} \\ (2k-1)\pi & \text{减弱. } k \text{ 级暗纹.} \end{cases}$$

相干光初相位相同时 $\Delta\varphi = \frac{2\pi}{\lambda} \delta$

$$\delta = \begin{cases} k\lambda & \text{加强. } k \text{ 级明纹.} \\ (k-\frac{1}{2})\lambda & \text{减弱. } k \text{ 级暗纹.} \end{cases}$$

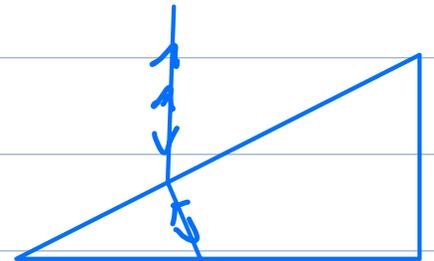
双缝干涉



$$\sin\theta \approx \tan\theta \approx \frac{x}{D}$$

$$\delta = d \sin\theta = \frac{x d}{D}$$

劈尖膜干涉



$$\delta = 2ne + \frac{\lambda}{2} \quad n \text{ 为介质 } e \text{ 为薄膜厚度}$$

牛顿环. $e = \frac{r^2}{2R}$

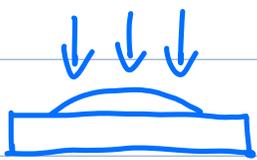
eg 1. 双缝 $d = 5 \times 10^{-4} \text{ m}$ $D = 3 \text{ m}$ $n = 1.5$ $e = 1 \times 10^{-5} \text{ m}$.

$$\delta = r_2 - r_1 - (n-1)e = \frac{\lambda d}{D} - (n-1)e.$$

$$\Delta\varphi = \frac{2\pi}{\lambda} \delta = 2k\pi \quad \delta = k\lambda. \quad x = \frac{D}{d} [(n-1)e + k\lambda].$$

当 $k=0$ 时. 中央明纹. $x_0 = \frac{D}{d} (n-1)e = 0.03 \text{ m}.$

eg 2. 油滴 $n=1.20$. 平玻璃片 $n=1.52$. 波长 $\lambda=600 \text{ nm}$ 垂直射.



油滴边缘处是亮纹. (两半波损失抵消).

$$2ne = 4\lambda. \quad e = \frac{2\lambda}{n} = 1 \times 10^{-6} \text{ m}.$$

eg 3. 白色平行光 $\lambda \in [400, 760] \text{ nm}$. 双缝间距 $d=0.25 \text{ mm}$.

屏幕距离 $D=0.5 \text{ m}$. 求第一级. 第五级彩色明纹带宽度.

$$\frac{\lambda d}{D} = k\lambda \quad \Delta x = \frac{k\lambda \cdot D}{d} \quad \Delta\lambda = 3.6 \times 10^{-7} \text{ m}.$$

$$k=1 \text{ 时 } \Delta x = 7.2 \times 10^{-6} \text{ m}. \quad k=5 \text{ 时 } \Delta x = 3.6 \times 10^{-3} \text{ m}$$

eg 4. 两块 $n=1.75$ 的玻璃砖形成空气劈尖. 一端厚度 0. 一端 $2 \times 10^{-3} \text{ cm}$.
 $\lambda=700 \text{ nm}$ 的平行单色光垂直射入. 干涉形成明纹条数.

$$\delta = 2ne + \frac{\lambda}{2} = k\lambda. \quad k_{\min}=1.$$

$$2nd = (k - \frac{1}{2})\lambda. \quad \text{此外 } n \text{ 为空气折射率. } k=57.6$$

∴ 明纹条数 57 条.

eg 5. $\lambda=650 \text{ nm}$ 的红光垂直照射到劈形液膜上. 膜的折射率 $n=1.33$.

液面两侧为同种媒质. 则第一条明纹对应膜厚度 —.

明纹间距 6 mm . 则第一条明纹到边缘距离为 —.

$$\delta = 2ne + \frac{\lambda}{2} = \lambda, \quad e = \frac{\lambda}{4n} = 1.22 \times 10^{-7} \text{ m}$$

$$e=0 \quad \delta = \frac{\lambda}{2} \text{ 为暗纹. } \Delta x = \frac{d}{\lambda} = 3 \text{ mm}$$

eg 6. λ 波长垂直射入牛顿环装置. 观察从空气膜上下表面反射的先形成的牛顿环. 使平凸透镜缓慢向上移动. 透镜顶点与平面从接触到相距 d 移过视场中某点的条纹数为

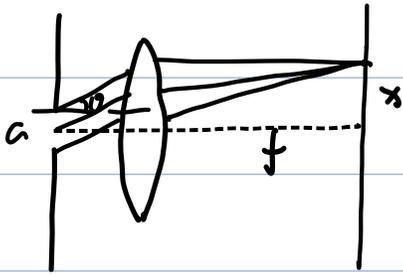
$$\delta = 2ne + \frac{\lambda}{2} = k\lambda, \quad k = \frac{2ne}{\lambda} + \frac{1}{2}.$$

$$\delta' = 2n(e+d) + \frac{\lambda}{2} = k'\lambda, \quad k' = \frac{2n(e+d)}{\lambda} + \frac{1}{2}$$

$$k' - k = \frac{2nd}{\lambda}, \quad n=1 \text{ (空气折射率). 移过条纹数 } \frac{2d}{\lambda}.$$

单缝衍射. 平行光垂直入射单缝.

a : 缝宽. θ : 光线相对缝法线偏转角 f : 透镜焦距. x : 条纹中心距.



$$\theta \approx \sin\theta \approx \tan\theta \approx \frac{x}{f}$$

$\theta = 0$ 中央明纹. 亮度最高.

$$a \sin\theta = \pm k\lambda \quad \text{暗纹中心. } k \neq 0$$

$$a \sin\theta = \pm (k + \frac{1}{2})\lambda \quad \text{明纹中心 } k \neq 0.$$

* 如果不是垂直入射. 有夹角 i . 则 $\sin\theta$ 修改为 $\sin\theta + \sin i$.

eg1. 波长 λ 平行单色光射一宽 a 的单缝. 焦距 f 的透镜紧靠后求中央明纹宽度. 即求两条一级暗纹间距.

$$a \sin\theta = \pm k\lambda. \quad k=1. \quad \sin\theta = \pm \frac{\lambda}{a}$$

$$\text{由几何关系知 } \sin\theta = \frac{x}{f}. \quad \therefore x = \pm \frac{\lambda f}{a} \quad \Delta x = \frac{2\lambda f}{a}.$$

光栅衍射. (光栅: 多个规律排列的单缝.)

a : 透光缝宽. b : 不透光缝宽 d : 光栅常数 $d = a + b$

主极大明纹.

光栅方程 $d \sin\theta = \pm k\lambda$. θ 处可以看见明纹.

由于 $\sin\theta \in [-1, 1]$. k 只能取有限数. 主极大明纹个数有限.

缺极

某些衍射角 θ 同时满足光栅方程和单缝衍射暗纹条件.

此时原定的主极大就会变成暗纹 \Rightarrow 缺极.

$$\begin{cases} a \sin\theta = k_1 \lambda \\ d \sin\theta = k_2 \lambda \end{cases} \quad \frac{k_1}{k_2} = \frac{a}{d}. \quad k_1, k_2 \text{ 均为 } N^{\circ} \text{ 时 } k_2 \text{ 为缺极.}$$

光栅常数为本值. 在某级恰好能分辨两谱线平均波长下. 与波长差比值

$$R = \frac{\bar{\lambda}}{\Delta\lambda} = kN.$$

$$5.9 \times 10^{-7} \text{ m}.$$

eg2. 每毫米500条栅纹的衍射光栅观察钠光谱线 $\lambda = 590 \text{ nm}$.

$a:b = 1:2$. 平行光垂直入射能看到哪些光谱线? 30° 时?

$$1) d = 1/500 = 2 \times 10^{-3} \text{ mm} = 2 \times 10^{-6} \text{ m}.$$

$$d \sin \theta = k\lambda. \quad \therefore k = \{-3, -2, -1, 0, 1, 2, 3\}$$

缺级 $\frac{a}{d} = \frac{1}{2} = \frac{k_1}{k_2}$. $\therefore k \neq \pm 3$. 缺级删去. 剩下 $k = 0, \pm 1, \pm 2$.

$$2) d(\sin \theta + \sin \gamma) = k\lambda. \quad k = \{-1, 0, 1, 2, 3, 4, 5\}$$

eg3. 单色光垂直射到8000条刻线每厘米的光栅上. 白光垂直照射时哪些光能观察到第二级谱线.

$$d = \frac{10^{-2}}{8 \times 10^3} = 1.25 \times 10^{-6} \text{ m} \quad \text{当 } k=2 \text{ 时. } d=2\lambda. \quad \lambda = 625 \text{ nm}$$

\therefore 能看到二级谱线的范围是 $400 \text{ nm} \sim 650 \text{ nm}$.

eg4. 一光栅. 白光垂直照射时. $\lambda = 720 \text{ nm}$ 的红光在衍射角 30° 方向上存在二级谱线. 且该级能分辨红光附近的最小波长差

$\Delta\lambda = 0.05 \text{ nm}$. 30° 方向上不存在可见光谱线及其他主极大.

1) 光栅常数. 2) 总缝数 3) 可能及最小缝宽.

$$1) d \sin \theta = k\lambda. \quad d = \frac{2 \times 7.2 \times 10^{-7}}{\frac{1}{2}} = 2.88 \times 10^{-6} \text{ m}.$$

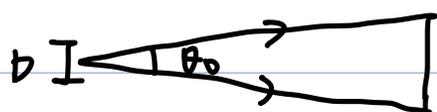
$$2) R = \frac{\lambda}{\Delta\lambda} = kN. \quad N = \frac{720}{0.05} \cdot \frac{1}{2} = 7200.$$

$$3) d \sin \theta = k'\lambda'. \quad \lambda' = \frac{1440}{k'} \quad k' = 3. \quad \lambda' = 480 \text{ nm}$$

$$\frac{a}{d} = \frac{k}{k'} \quad a = \frac{kd}{k'} \quad k=1 \text{ 或 } 2$$

$$a = 960 \text{ nm 或 } 1920 \text{ nm.} \quad a_{\min} = 960 \text{ nm}$$

圆孔衍射 (最小分辨角)



恰好能分辨. $\theta_{\min} = 1.22 \frac{\lambda}{D}$.

D : 圆孔直径 (即人眼). $\theta < \theta_{\min}$ 时无法分辨.

eg 5. 人瞳孔直径 3 mm. 视觉最敏感波长 550 nm

$$\text{最小分辨角 } \theta_{\min} = 1.22 \frac{\lambda}{D} = 1.22 \frac{5.5 \times 10^{-7}}{3 \times 10^{-3}} = 2.24 \times 10^{-4} \text{ rad.}$$

黑板上 " = " 相距 2 mm. 笔在离黑板多远处能看清.

$$\frac{l}{2} = \tan \theta \approx \frac{l}{2} = \frac{1 \times 10^{-3}}{d} = 1.12 \times 10^{-4}. \quad d = \frac{10}{1.12} = 8.9 \text{ m.}$$

X 射线在晶体上衍射.

θ : X 光与晶面间的掠射角. d 相邻晶面间距.

能产生强反射的角度条件: $2d \sin \theta = k\lambda$. $k=1, 2, \dots$

eg 6. 一束 X 射线含 0.095 nm 到 0.13 nm 中各种波长. $\theta = 45^\circ$

晶格常数 $d = 0.275 \text{ nm}$. 求强反射 X 射线的波长.

$$2d \sin \theta = k\lambda. \quad k = \frac{\sqrt{2} \cdot 0.275}{\lambda} \quad \text{或 } N^{\pm}.$$

$$\lambda_1 = 0.1296 \text{ nm} \quad \lambda_2 = 0.0972 \text{ nm.}$$

光束的分类. 线偏振光. 自然光. 部分偏振光.

理想偏振片: 平行光全通过. 垂直完全不通过.

马吕斯定律:

$$I = \frac{1}{2} I_0 \quad \text{自然光}$$

$$I = I_0 \cos^2 \theta \quad \text{线偏振} \quad \theta \text{ 为原振动方向与指定方向}$$

$0 \sim 90^\circ$.

夹角

eg. I_0 自然光穿过三个偏振片 P_1, P_2, P_3 . 光强 $I_3 = \frac{1}{8} I_0$.

P_1, P_3 偏振化方向相互垂直. 如何旋转 P_2 使 $I_3 = 0$

$$I_1 = \frac{1}{2} I_0 \quad I_2 = \frac{1}{2} I_0 \cos^2 \theta \quad I_3 = \frac{1}{2} I_0 \cos^2 \theta \cos^2 (90^\circ - \theta)$$

$$I_3 = \frac{1}{2} I_0 \cos^2 \theta \sin^2 \theta = \frac{1}{8} I_0 \sin^2 2\theta \Rightarrow \sin^2 2\theta = 1. \quad \theta = \frac{\pi}{4}$$

\therefore 将 P_2 旋转 45° 即可.

eg. 一束自然光与线偏振光混合通过一旋转的偏振片

$$I_{\max} / I_{\min} = 5 \quad \text{求 } I_{\text{自}} / I_{\text{线}}$$

$$I_{\max} = \frac{1}{2} I_{\text{自}} + I_{\text{线}} \quad I_{\min} = \frac{1}{2} I_{\text{自}}$$

$$\frac{1}{2} I_{\text{自}} + I_{\text{线}} = \frac{1}{2} I_{\text{自}} \cdot 5 \quad \therefore I_{\text{自}} / I_{\text{线}} = \frac{1}{2}$$

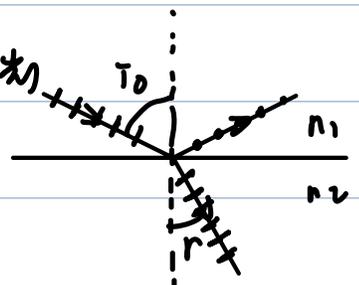
布儒斯特定律.

入射角 i_0 . 反射光振动方向垂直入射面 (线偏振光)

折射光成为最大偏振化程度的部分偏振光

$$\begin{cases} \tan i_0 = \frac{n_2}{n_1} \\ i_0 + r = 90^\circ \end{cases}$$

用于制造极佳的偏振光.



eg 3. 三种介质折射率 $n_1 = 1.00$, $n_2 = 1.43$, n_3 . 界面相互平行

一束自然光由介质 I 中入射. 在两个界面上反射光都是偏振光.

求入射角 r . 折射率 n_3 .

$$\tan i_0 = \frac{n_2}{n_1} \quad i_0 = \arctan 1.43 = 55.03^\circ$$

$$r_1 = 34.97^\circ \quad \tan r_1 = \frac{n_3}{n_2} \quad n_3 = 1.00$$

双折射

光线射到各向异性晶体时会分成偏振方向不同的两束光.

寻常光 (o 光) 服从折射定律 非寻常光 (e 光) 不服从

光轴: 沿此方向入射不发生双折射.

光轴与入射面平行时, o 光 e 光与主平面重合, 都在入射面内.

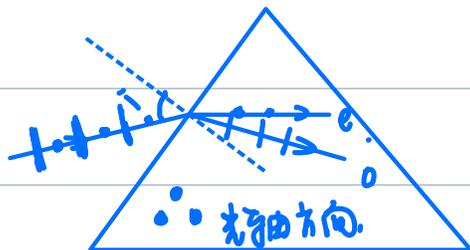
振动方向 \perp 光轴, 传播速度正常, 折射率 n_o .

振动方向 \parallel 光轴, 传播速度达最值, 折射率 n_e (主折射率).

正晶体 $n_e > n_o$ 负晶体 $n_o > n_e$.

eg 4. 方解石 (负晶体) 切成三角形棱镜, 光轴方向如图. 自然光以 i 角入射并发生双折射. 画出 o 光, e 光光路及振动方向.

$n_o > n_e$.



波片: 厚度均匀, 两表面与光轴平行的晶体片, 线偏振光正入射表面

偏振方向与光轴夹角为 θ .

① 沿光轴方向分为 o 光和 e 光.

② 相位差分析. 光程差 $\delta = |n_o - n_e| d$ 相位差 $\Delta\varphi = \frac{2\pi}{\lambda} |n_o - n_e| d$.

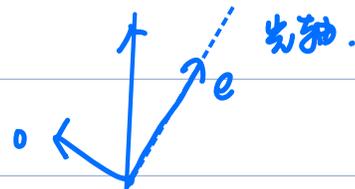
对确定的 λ . 要产生特定的 $\Delta\varphi$. d 就取特定值.

$\frac{1}{4}$ 波片 $\frac{\lambda}{4}$. $\frac{1}{2}$ 波片 $\frac{\lambda}{2}$. 确定波长的.

eg 5. $\lambda = 589.3 \text{ nm}$ 的线偏振光垂直入射到石英晶片上. 晶片光轴垂直于表面. 入射光与光轴夹角 30° . $d = 4092.36 \text{ nm}$. $n_e = 1.553$. $n_o = 1.541$.

1) o 光和 e 光的相位差.

2) 出射光中 o 光和 e 光的强度之比.



1) $\delta = |n_o - n_e| \cdot d = 49.10832 \text{ nm}$. $\Delta\varphi = \frac{2\pi}{\lambda} \delta = 2\pi \frac{\delta}{\lambda} \approx \frac{\pi}{6}$.

2) $I_e = I_0 \cos^2 30^\circ = \frac{3}{4} I_0$ $I_o = I_0 \cos^2 60^\circ = \frac{1}{4} I_0$ $\frac{I_o}{I_e} = \frac{1}{3}$

eg 6. 偏振化方向正交的偏振片 P_1, P_2 中间放置一块方解石晶片. 光轴平行晶体表面. 与 P_1 偏振化方向夹角为 30° $n_e = 1.658$ $n_o = 1.486$

1) 强度 I 的自然光垂直射入 P_1 . 晶片透射出的 o 光. e 光的强度.

2) $\lambda = 400 \text{ nm}$ P_2 后无透射光. 晶片至少多厚.

1) $I \rightarrow I_1 = \frac{1}{2} I$ $I_e = I_1 \cos^2 30^\circ = \frac{3}{8} I$. $I_o = I_1 \cos^2 60^\circ = \frac{1}{8} I$.

2) $A_e = \sqrt{3} A_o$ $A_e \cos 60^\circ = A_o \cos 30^\circ$. 无额外相位差.

$\Delta\varphi = \frac{2\pi}{\lambda} |n_e - n_o| d = 2k\pi$. $k \in \mathbb{N}^+$.

$d = \frac{k\lambda}{|n_e - n_o|} \geq \frac{\lambda}{|n_e - n_o|} = \frac{400}{0.172} = 2325.6 \text{ nm}$

偏振光通过晶片后两个方向的振动产生相位差。两个振动产生特殊偏振光。

eg 7. 入线偏振光穿过一晶片。入射光振动方向与晶片光轴夹角 45° 。

晶片为 $\lambda/4$ 晶片。出射光为——偏振光。若要使圆偏振光出射。晶片厚度 d 应为——。 n_o, n_e 已知。

$$\Delta\varphi = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \pi. \quad \text{振动合成以后为线偏振光.}$$

$$\Delta\varphi = \frac{\pi}{2} \frac{2\pi}{\lambda} (n_o - n_e) d. \quad d_{\min} = \frac{\lambda}{4|n_o - n_e|}.$$

* 会聚透镜 $f > 0$ 发散透镜 $f < 0$.

① 薄透镜方程. $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$

s 与入射光线同侧为正. 反之则为负.

s' 与出射光线同侧为正. 反之则为负.

② 横向放大率. $m = \frac{y'}{y} = -\frac{s'}{s}$

③ 磨镜者公式. $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ R 为透镜球面半径. 平面时取 ∞ .

eg. 平凸透镜一侧 R . 折射率 n . 求 f . 选取平面入射. $R_1 = \infty$. $R_2 = -R$

$$\frac{1}{f} = (n-1)\left(\frac{1}{\infty} - \frac{1}{-R}\right) = \frac{n-1}{R} \quad f = \frac{R}{n-1}$$

eg. 发散透镜 $f = 15 \text{ cm}$ $s = 30 \text{ cm}$ 处有 $y = 12 \text{ cm}$ 的物体

求 s' . y' . m .

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad s' = -10 \text{ cm}$$

$$m = -\frac{s'}{s} = \frac{1}{3} \quad m = \frac{y'}{y} \quad y' = 4 \text{ cm}$$

* 放大镜 $m_0 = \frac{25 \text{ cm}}{f}$.

* 显微镜 $M = m \cdot m_0 = -\left(\frac{s}{f_0}\right)\left(\frac{25 \text{ cm}}{f_e}\right) \quad s = f_0 + \Delta. \quad \Delta = |F_2 F_1'|.$

黑体辐射.

单色辐出度 $M_\lambda(T)$ 单位时间 单位表面积 $\lambda \rightarrow \lambda + d\lambda$ 辐射能 dM_λ .

$$M_\lambda(T) = \frac{dM_\lambda}{d\lambda}$$

辐射出射度 单位面积全波长辐射能

$$M(T) = \int_0^\infty M_\lambda(T) d\lambda.$$

吸收. 反射. 透射.

吸收系数 α . 反射系数 r . ($\lambda, T = \text{元函数}$) 不透明物体 $\alpha + r = 1$.

绝对黑体. $\alpha_B(\lambda, T) = 1$

$$\text{基尔霍夫定律 } \frac{M_\lambda(T)}{\alpha(\lambda, T)} = M_{B\lambda}(T)$$

$$\text{Stefan-Boltzmann } M_B(T) = \int_0^\infty M_{B\lambda}(T) d\lambda = \sigma T^4.$$

Wien 位移. $T\lambda_m = b$

$$\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 \quad b = 2.898 \times 10^{-3} \text{ m} \cdot \text{K}$$

eg1. 黑体在某温度辐射出射度 $5.7 \times 10^4 \text{ W/m}^2$

求温度 T . 光谱峰值波长 λ_m .

$$5.7 \times 10^4 = 5.67 \times 10^{-8} \times T^4 \quad T = 1.001 \times 10^3 \text{ K.}$$

$$\lambda_m \cdot 1.001 \times 10^3 = 2.898 \times 10^{-3} \quad \lambda_m = 2.898 \times 10^{-6} \text{ m}$$

光电效应.

$$\text{光子: } E = h\nu \xrightarrow{E=mc^2} m = \frac{h\nu}{c^2} = \frac{h}{c\lambda} \xrightarrow{P=mc} P = \frac{h\nu}{c} = \frac{h}{\lambda}.$$

能量 质量 动量.

逸出功 A . 最大初动能 E_{km} . $h\nu = E_{km} + A$.

截止电压 U_a $E_{km} = e|U_a|$ 此时光电流为 0.

截止频率 (红限频率) ν_0 $A = h\nu_0$

光强 I $I = n h \nu$ n : 光子数 \Rightarrow 饱和光电流.

eg2. ν_1, ν_2 的单色光. 同一金属光电效应. (ν_0) $|U_{a1}| = 2|U_{a2}|$

求 ν_1, ν_2 之间的关系.

$$\begin{cases} h\nu_1 - h\nu_0 = e|U_{a1}| \\ h\nu_2 - h\nu_0 = e|U_{a2}| \end{cases} \Rightarrow \begin{cases} h\nu_2 - h\nu_0 = 2(h\nu_1 - h\nu_0) \\ \nu_2 + \nu_0 = 2\nu_1 \end{cases}$$

eg3. $\nu_1 > \nu_2 > \nu_0$ 照射一光电管 光强相同时. 问光电流大小关系.

$$n_1 h \nu_1 = n_2 h \nu_2 \quad n_1 < n_2 \quad \text{光电流 } I_1 < I_2$$

康普顿效应.

能量: $h\nu_0 + m_e c^2 = h\nu + m c^2$ m_e 静止质量.

动量: $\begin{cases} \frac{h\nu_0}{c} = \frac{h\nu}{c} \cos\varphi + m v \cos\theta \\ 0 = \frac{h\nu}{c} \sin\varphi - m v \sin\theta \end{cases}$ m 相对论质量.

波长改变与散射角 $\Delta\lambda = \lambda - \lambda_0 = \frac{h}{m_e c} (1 - \cos\varphi)$

$\frac{h}{m_e c} = 0.0024 \text{ nm}$ φ 可以到 180° .

电子动能 $E_k = h\nu_0 - h\nu$.

eg4. 0.5 MeV 的 x-ray. 发生康普顿散射. 反冲电子获得能量 0.1 MeV.

求波长改变量 $\Delta\lambda$ 与波长 λ_0 的比值.

$$\frac{hc}{\lambda_0} = 0.5 \text{ MeV} \quad \frac{hc}{\lambda} = 0.4 \text{ MeV} \quad \lambda / \lambda_0 = 1.25$$

$$\Delta\lambda / \lambda_0 = (\lambda - \lambda_0) / \lambda_0 = \lambda / \lambda_0 - 1 = 0.25$$

eg 5 康普顿散射实验 λ 射光波长 0.0030 nm . 反冲电子速度 $0.6c$.

求 1) 散射光子波长. 2) 散射光子散射角.

$$\frac{hc}{\lambda_0} - \frac{hc}{\lambda} = \frac{mc}{\sqrt{1-v^2/c^2}} \cdot c^2 - mc^2$$

$$\frac{1}{\lambda_0} - \frac{1}{\lambda} = 0.5 \frac{mcc}{h} \quad \lambda_0 = 0.0030 \text{ nm} \quad \frac{h}{mcc} = 0.00248$$

解得 $\lambda = 0.00436 \text{ nm}$.

$$\lambda - \lambda_0 = \frac{h}{mcc} (1 - \cos\varphi) \quad \cos\varphi = \dots \quad \varphi = 63^\circ$$

德布罗意波. $\lambda \rightarrow p \quad v \rightarrow E.$

$$E = mc^2 = h\nu \quad \left\langle \begin{array}{l} E = p^2/2m \\ p = mv = h/\lambda \end{array} \right. \rightarrow p = mv = h/\lambda.$$

eg. 静电子经过 U 加速后. $\lambda = 0.04 \text{ nm}$. 求 $U =$

$$eU = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} \quad U = \frac{h^2}{2me\lambda^2} = 943 \text{ V}.$$

eg. 一束动量 p 的电子经过宽度 a 的狭缝. 距狭缝 R 处屏上衍射最大宽度 d . 求 a .

$$\lambda = \frac{h}{p} \quad a \sin\theta = k\lambda \quad k = \pm 1 \quad (\text{求暗纹}), \quad \sin\theta = \frac{d}{2R}.$$

$$a = \frac{2R}{d} \cdot \frac{h}{p} = \frac{2Rh}{pd}$$

不确定性原理.

$$\Delta x \Delta p_x \geq \frac{h}{2} \quad \Delta E \Delta t \geq \frac{h}{2}$$

eg $\lambda = 500 \text{ nm}$. $\Delta\lambda = 10^{-4} \text{ nm}$ 的光沿 x 轴正向传播求 x 的不确定量.

$$p_x = \frac{h}{\lambda} \quad \Delta p_x = \frac{h\Delta\lambda}{\lambda^2} \quad \Delta x \Delta p_x = \Delta x \cdot \frac{h\Delta\lambda}{\lambda^2} \geq \frac{h}{4\pi}.$$

$$\Delta x \geq \frac{\lambda^2}{4\pi\Delta\lambda} = \frac{25 \times 10^{-14}}{4\pi \cdot 10^{-13}} = 0.2 \text{ m}.$$

eg. 电子处于某原子能态 $\Delta t = 10^{-8} \text{ s}$ 计算能量不确定量; 电子从此能态跃迁到基态对应

光子能量 3.37 eV . 求光子波长及其不确定量.

$$\Delta E \Delta t \geq \frac{h}{2} \quad \Delta E \geq \frac{h}{4\pi\Delta t} = 5.276 \times 10^{-27} \text{ J}$$

$$\lambda = \frac{hc}{E} = 366.5 \text{ nm}$$

$$\Delta\lambda = \frac{hc\Delta E}{E^2} \geq 3.56 \times 10^{-15} \text{ m}$$

波函数 $\Psi(x, y, z, t)$ 时空函数.

定态波函数 $\Psi(x, y, z)$ 可能是复数. $|\Psi|^2 = \Psi\Psi^*$ 为概率密度.

$\int \Psi\Psi^* = 1$ 归一化条件.

薛: $i\hbar \frac{\partial}{\partial t} \Psi(r, t) = \left[-\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) + \bar{V}_p \right] \Psi(r, t)$

- 维定态 $\frac{d^2\Psi}{dx^2} + \frac{2m}{\hbar^2} [E - \bar{V}_p] \Psi(x) = 0$

- 般定态 $\frac{d^2\Psi}{dx^2} + \frac{d^2\Psi}{dy^2} + \frac{d^2\Psi}{dz^2} + \frac{2m}{\hbar^2} [E - \bar{V}_p] \Psi = 0$.

- 维谐振 $\frac{d^2\Psi}{dx^2} + \frac{2m}{\hbar^2} [E - \frac{1}{2}m\omega^2 x^2] \Psi = 0$.

eg. 长为 L 的一维无限深势阱, 粒子在第一激发态 $\Psi(x) = A \sin \frac{2\pi x}{L}$. $0 < x < L$

1) 归一化常数 A . 2) 粒子分布概率密度函数 3) 最大概率处. 4) $0 \sim \frac{1}{3}$ 概率.

$$1) A^2 \int_0^L \sin^2 \frac{2\pi x}{L} = \frac{A^2}{2} \int_0^L (1 - \cos \frac{4\pi x}{L}) dx = \frac{A^2}{2} L = 1 \quad A = \sqrt{\frac{2}{L}}$$

$$2) f(x) = \Psi^2(x) = \frac{2}{L} \sin^2 \frac{2\pi x}{L}$$

$$3) \left. \begin{aligned} \sin \frac{2\pi x}{L} = 1 & \quad \frac{2\pi x}{L} = \frac{\pi}{2} + 2k\pi = \frac{1}{4}L + kL \\ = -1 & \quad \frac{2\pi x}{L} = -\frac{\pi}{2} + 2k\pi = \frac{3}{4}L + kL \end{aligned} \right\} \frac{1}{4} \text{ or } \frac{3}{4}L$$

$$4) \int_0^{1/3} \Psi^2(x) = \frac{1}{L} \int_0^{1/3} (1 - \cos \frac{4\pi x}{L}) dx = \frac{1}{L} \left(x - \frac{L}{4\pi} \sin \frac{4\pi x}{L} \right) \Big|_0^{1/3}$$
$$= \frac{1}{3} - \frac{1}{4\pi} (\sin \frac{4\pi}{3} - 0) = \frac{1}{3} + \frac{\sqrt{3}}{8\pi} = 0.4$$

势垒穿透 $T = e^{-2ka}$: 透射率.

$$k = \frac{1}{\hbar} \sqrt{2m(\bar{V}_p - \bar{E})} = \sqrt{\frac{8\pi^2 m (\bar{V}_p - \bar{E})}{\hbar^2}}$$

eg. 电子总能量 5.0 eV 势垒高度 6.0 eV 求 1% 穿透概率时厚度 a

$$k = \frac{1}{\hbar} \sqrt{2m(E_p - E)} = \frac{2\pi}{\hbar} \sqrt{2 \times 9.11 \times 10^{-31} \times 1.6 \times 10^{-19}} = 5.2 \times 10^9 \text{ m}^{-1}$$

$$\frac{1}{100} = e^{-2ka} \quad a = \frac{\ln 10}{k} = 4.5 \times 10^{-10} \text{ m}$$

角动量 $L = mvr = nh$ 方的整数倍。

$$\Rightarrow v = \frac{nh}{2m\pi r}$$

轨道半径. $\frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} = m \frac{v^2}{r} = m \left(\frac{nh}{2m\pi r} \right)^2 \cdot \frac{1}{r^3}$

$$\Rightarrow r = n^2 \frac{\epsilon_0 h^2}{\pi m e^2} \quad \frac{\epsilon_0 h^2}{\pi m e^2} = a_0 \text{ 玻尔半径. } r_n = n^2 \cdot a_0$$

$$v = \frac{nh}{2m\pi r} = \frac{nh}{2m\pi} \cdot \frac{1}{n^2} \cdot \frac{\pi m e^2}{\epsilon_0 h^2} = \frac{1}{n} \cdot \frac{e^2}{2\epsilon_0 h}$$

能量量子化. $E_n = -\frac{1}{2} m v^2 = -\frac{1}{n^2} \cdot \frac{m e^4}{8 \epsilon_0^2 h^2} = \frac{1}{n^2} E_0$

$$E_0 = -\frac{m e^4}{8 \epsilon_0^2 h^2} = -13.6 \text{ eV}$$

eg. 基态氢原子被 12.09 eV 的光子激发. 其电子轨道半径变为玻尔半径的一倍。

$$E_n = -13.6 + 12.09 = -1.51 \text{ eV} = \frac{1}{n^2} E_0 \quad n=3$$

$$r_n = n a_0 = 3 a_0$$

氢原子光谱. 定态 \rightarrow 定态 (吸收/发射光子).

$$\text{高利低. } \frac{hc}{\lambda} = E_1 - E_2 = -E_0 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$\frac{1}{\lambda} = \frac{-E_0}{hc} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \quad R: \text{里德伯常量}$$

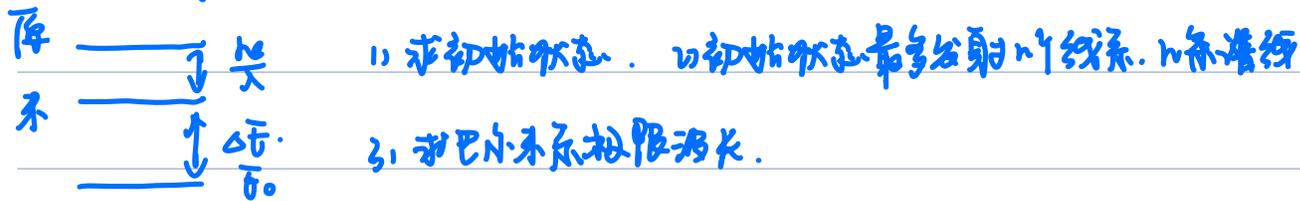
$n_1=1$: 莱曼系. 紫外.

$n_1=2$: 巴尔末. 可见.

$n_1=3$
 $n_1=4$
 $n_1=5$ } 红外

n 能级. 谱线 $\frac{n(n-1)}{2}$ 条. 线系 $(n-1)$

eg. 氢原子从初始状态跃迁到激发能 $\Delta E = 10.19 \text{ eV}$ 的状态. 发出 $\lambda = 486 \text{ nm}$ 的光子.



$$1) E_n = E_0 + \Delta E + \frac{hc}{\lambda} = -0.85 \text{ eV} \quad n=4.$$

$$2) \text{ 线系: } n-1=3 \quad \text{谱线 } \frac{n(n-1)}{2} = 6$$

$$3) \frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n^2} \right) \quad n \rightarrow \infty = \frac{R}{4}.$$

$$\lambda = \frac{4}{R} = 364.6 \text{ nm}.$$

量子数.

主量子数 n $0, 1, \dots, \infty$ 电子能量 $E_n = -\frac{E_0}{n^2}$.

角量子数 l $0, 1, \dots, (n-1)$ 角动量 $L = \sqrt{l(l+1)} \hbar$.

磁量子数 m_l $0, \pm 1, \dots, \pm l$ z 轴方向角动量 $L_z = m_l \hbar$.

eg. 氢原子 $3d$ 电子. 轨道角动量与 z 轴最小夹角.

$$n=3 \quad l=2 \quad L = \sqrt{6} \hbar \quad (L_z)_{\max} = 2 \hbar.$$

$$(\cos \theta)_{\max} = \frac{\sqrt{6}}{3} \quad \theta_{\min} = \arccos \frac{\sqrt{6}}{3}.$$

电子自旋. S 态银原子在磁场中分成两束.

$$m_s = \pm \frac{1}{2}$$

概率密度.

$|\Psi_{n,l,m_l}(r, \theta, \varphi)|^2$ 表示电子出现在 (r, θ, φ) 的概率密度.

$r^2 |R_{n,l}(r)|^2$ 表示出球在 r 处的径向概率密度.

eg. $\psi_{n,l,m_l}(r,\theta,\varphi) = \psi_{2,1,1}(r,\theta,\varphi) = C r e^{-\frac{r}{2a_0}} \sin\theta e^{i\varphi}$

1) 求此状态下氢原子的 E 和 L

2) 同样 n 下可能的状态数.

3) 径向概率密度最大处.

1) $E = \frac{E_0}{n^2} = \frac{E_0}{4} = -3.4 \text{ eV}$ $L = \sqrt{2} \hbar$.

2) $n=2$ $l=0, 1$ $m_l (2l+1)$ 种 m_s 2 种.

$$N = 2 \sum_{l=0}^1 (2l+1) = 8$$

3) $F(r) = r^2 |R(r)|^2 = r^2 \cdot r^2 \cdot e^{-r/a_0} = r^4 \cdot e^{-r/a_0}$

$$\frac{dF}{dr} = (4r^3 - r^4 \cdot \frac{1}{a_0}) e^{-r/a_0} = 0 \quad r = 4a_0.$$